Engineering

Developing an understanding of square numbers

Content and alignment

Understanding square numbers, and the standard notation, is essential in the engineering trades for several reasons. First, square numbers are a common feature within geometry, and second, finding and using square numbers is essential to a range of more complex formulae and methods used in the engineering field. For example, understanding square numbers is essential before learners can understand the Pythagoras Theorem. In fact, misconceptions about square numbers are the main reasons learners struggle with the Pythagoras Theorem.

Intent

The intent of this resource is to provide learners with a visual example of square numbers, and an opportunity to discuss the concept and dispel misconceptions. For example, many learners believe that the mathematical notation 4^2 is equivalent to 8 rather than 16. The standard notation is often interpreted as 4 x 2. In some cases, examples such as 2^2 perpetuate the problem as the answer is 4. This resource follows a sequence that ensures learners develop a clear understanding of the concept.

Sequence

1. Introduce square numbers
2. Explore square numbers
3. Understand the exponent notation

1. Introduce square numbers

Ensuring learners make the link between 'square' numbers and an actual square is important. In this step learners define a square as a flat shape (a plane) with four equal straight sides.

Step one: Ask your learners in a whole-class setting what 'exactly' a square is. Allow learners to share ideas, gradually adding more details. For example, a discussion such as this could be held:

Learner: "It's a shape that has four equal sides."

Tutor: "Are all the sides straight?"

Learner: "Yes. So a square is a shape that has four equal *straight* sides."

Tutor: "Is the surface flat?"

Learner: "Yes. A square is a *flat* shape that has four equal *straight* sides."

Tutor: "Great, so a square has a few requirements that many people are unaware of. It has to be a flat shape (a plane) that has four equal, straight sides."

Step two: Draw or project a grid onto the whiteboard. Ask the learners:

How do you draw a square on a grid or an array?

The learners often respond with comments such as, "Just make sure it's the same on both sides."

Model a square on the grid and make the point that a square has an equal number of columns and rows. Understanding this will lead to better understanding of units of measure and square numbers that are not whole numbers.

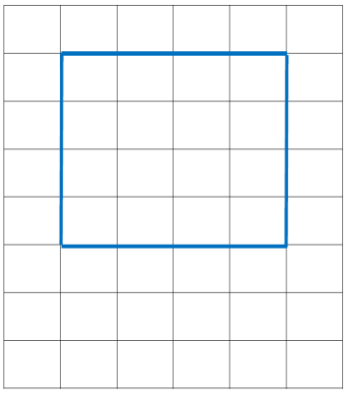

Fig 1.

2. Explore square numbers

In this step learners learn about square numbers and how they are calculated.

Step one: Hand out the dot array on page 6. Ask learners to mark out squares using the dots. See Figure 2 for an example.

The challenge: How many squares can be made, without the squares crossing each other, and without repeating a square of the same size? Give learners a few minutes only.

In most cases, the first attempt by learners is disorganised.

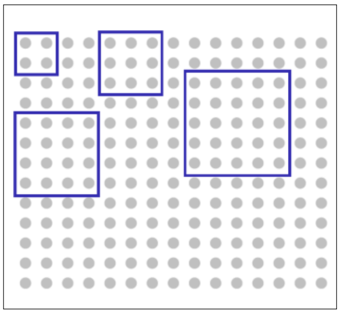

Fig. 2

Step two: After a few minutes ask the learners how many squares they managed to make on their sheet. Record the highest number on the board. Then ask the learners what size squares they managed to make. Write these up also.

In most cases learners would want to repeat the activity to beat their score. This leads to more organised thinking about how the squares may fit.

Step three: Ask learners to do the following:

Count the dots in each square and order them from smallest to largest.

This is best done by using a table.

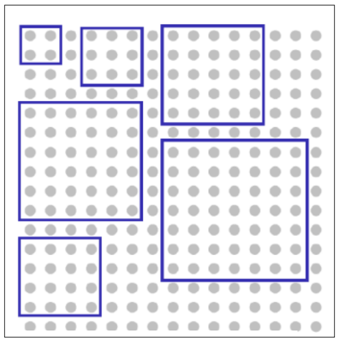

Fig. 3

	Smallest						**Largest**
Total dots	4	9	16	25	36	49	64

Step four: Have the learners count and record the dots on one side of each square and include these under the total number of dots.

	Smallest						**Largest**
Total dots	4	9	16	25	36	49	64
Dots on each side	2	3	4	5	6	7	8

Step five: Ask learners to discuss the connections between the length of the sides and the total number of squares. For example: Is there a quick way to work out how many dots there are in each square?

In most cases there will be some learners who will notice that the total number of squares is equal to the length multiplied by itself.

Learners may say: "You take one of the sides and times it by itself."

Restate this and write it on the board: The total area of a square is equal to any side multiplied by itself. If we write this as a sum, you can see that the number always appears twice.

2 x 2, 3 x 3, 4 x 4, 5 x 5 etc. This is an introduction to the exponent (x^2).

3. Understand the exponent notation

Step one: Use a data projector to project the dot array onto the whiteboard (this can be found in Engineering: Dot array, grid and coordinate sheets).

Draw a line of three dots long (see figure 4). Ask the learners: If we turn this line into the side of a square, how large will our square be? Allow the learners to answer and then draw in the square (see figure 5).

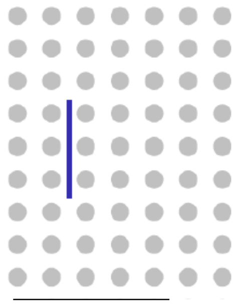

Fig. 4

Step two: Ask the learners:

What is the easiest way to calculate the total amount of dots?

Most learners will agree that multiplying the sides is the fastest way.

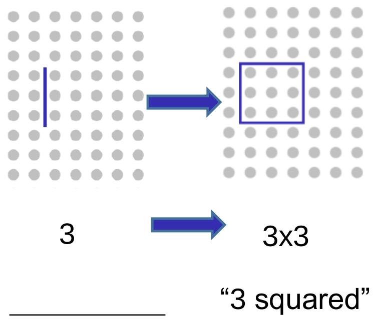

Fig. 5

Step three: Here is where you make the explicit connection between the mathematical notation (3^2) and the actual act of making a square.

As you draw a line on the whiteboard, ask the learners how long the other sides of the square will be. Draw the square and ask how many dots will be inside the square.

Follow this by explaining that you took a line and calculated its size as a square. Then explain to the learners that when the exponent is a ‘2’ it is pronounced ‘squared’ because the exponent is squaring the side.

The exponent $^{(2)}$

is pronounced ‘squared’ because it does the job of turning a length into a side of a square, and calculating the area.

Step four: Hand out a new dot array. Write several square numbers on the board such as:

- 3^2
- 4^2
- 5^2
- 2^2

Have the learners make the squares on the dot array and calculate the total area of the squares.

Finish with a class discussion.

Follow up: A primary point of this sequence is that learners develop their understanding of 'squared numbers' beyond simply thinking that the exponent means "You times it by itself". The deeper understanding will have the added bonus of supporting learners understanding of the concept of 'the square root' (√) as determining the length of one side of the square.

Additionally, the use of square numbers ought to be contextualised as quickly as possible. This resource may be used as an introduction to the Pythagoras Theorem.

Summary

This resource has three parts to it. First, the sequence begins with a definition of what a square is, as this sets learners up to make connections between square numbers and actual squares. Second, learners make connections between square numbers and their roots, in the context of actual squares. Instead of only seeing exponents and squares in the context of digits, learners are encouraged to see them as representative of real objects. Finally, the mathematical notation is made explicit and connected to the act of 'squaring' an actual length.

Dot array