



Applying mathematics knowledge to industry orientated problems in classroom teaching

Ziming Tom Qi, Otago Polytechnic

May 2015

Author

Associate Professor Ziming Tom Qi

2013 Project team

Associate Professor Ziming Tom Qi (Project Leader & Principal Investigator), Richard Nyhof, Dr. Matt King and Dr. Najif Ismail, Otago Polytechnic

2014 Project team

Associate Professor Ziming Tom Qi and Dr. Mark Harmer, Otago Polytechnic

Dr. Ken Louie and Debbie Hogan, Waikato Institute of Technology

Daphne Robson, Christchurch Polytechnic Institute of Technology

Frank Cook, Wellington Institute of Technology

ACKNOWLEDGEMENTS

This report is based on contributions from the project team as detailed below:

In 2013, Richard Nyhof (Otago Polytechnic) provided the question list to industry leaders. He assisted with the Otago Polytechnic Ethics Application. He also assisted with the implementation of the industry case studies, data collection and analysis. Dr Matt King (Otago Polytechnic) assisted with the industry case studies in mechanical engineering and Dr Najif Ismail (Otago Polytechnic) assisted with the industry case studies in civil engineering.

In 2014, Dr Ken Louie (Waikato Institute of Technology) provided the formative test that was implemented at the four Institutes of Technology and Polytechnics included in this project. He assisted with the implementation of the industry case studies at Waikato Institute of Technology and the data collection. He also assisted with gaining ethics approval from Waikato Institute of Technology. Debbie Hogan assisted with gaining ethics approval from Waikato Institute of Technology.

In 2014, Daphne Robson (Christchurch Polytechnic Institute of Technology) assisted with the implementation of the industry case studies at Christchurch Polytechnic Institute of Technology, data collection and data analysis. She also assisted with gaining ethics approval from Christchurch Polytechnic Institute of Technology.

In 2014, Frank Cook (Wellington Institute of Technology) assisted with the implementation of the industry case studies at Wellington Institute of Technology and data collection. He also assisted with gaining ethics approval from Wellington Institute of Technology.

In 2014, Dr Mark Harmer (Otago Polytechnic) assisted with the implementation of the industry case studies at Otago Polytechnic and data collection.

I would also like to take this opportunity to thank Otago Polytechnic, Waikato Institute of Technology, Wellington Institute of Technology, and Christchurch Polytechnic Institute of Technology for supporting this study. Special acknowledgements are made to:

Jenny Aimers, Research Coordinator, Otago Polytechnic; Alistair Regan, Research and Enterprise Director, Otago Polytechnic; Phil Ker, Chief Executive, Otago Polytechnic; John Findlay, Head of School of Architecture Building and Engineering, Otago Polytechnic.

I would particularly like to express my thanks and acknowledge our debt to all those organisations and individuals who have contributed to the preparation of this report. I hope contributors will recognize that I have done my best to accurately reflect the variety of views and the wealth of information which were so generously provided.

Finally, I would like to acknowledge Bridget O'Regan, Southern Hub Regional Manager, Ako Aotearoa Southern Hub, for her support and unfailingly professional yet friendly advice. It is much appreciated.

Contents

ACKNOWLEDGEMENTS	i
EXECUTIVE SUMMARY	2
INTRODUCTION	4
PROJECT AIMS	5
LITERATURE REVIEW.....	5
METHODOLOGY.....	6
RESULTS AND FINDINGS.....	7
Summary of findings.....	7
Analysis of Stage 1 results	7
Survey of engineering industry leaders	7
Teaching using industry samples	7
Nature of the industry oriented questions	15
Survey of mathematics lecturers	16
CONCLUSION AND RECOMMENDATIONS	17
REFERENCES	18
Appendix 1: Question list for focus group with industry leaders.....	20
Appendix 2: Industry sample for civil engineering.....	30
Appendix 3: Industry sample for mechanical engineering.....	32
Appendix 4: 2014 Formative test trialed at the four Metro Group ITPs	33

EXECUTIVE SUMMARY

This project aimed to identify and test an industry oriented maths approach within four Institutes of Technology and Polytechnics' Bachelor of Engineering Technology (BEngTech) programmes. The research question was: can BEngTech students apply the purely theoretical maths taught in the current course to industry related maths problems?

The project was completed in two phases. Phase 1 was conducted by the research team at Otago Polytechnic where 22 students were registered to the maths class and 16 of these students attended the test for the research. Real life industry related maths problems were collected from potential employers of BEngTech students and surveyed for maths knowledge. A workshop was offered to the first year BEngTech maths students on industry related maths problems in 2013. A comparison of the performance of students in theoretical maths problems was undertaken on the maths results from the 2012 and 2013 student cohorts.

Phase 2 was conducted in 2014. The research methodology was redesigned to test a different cohort of first year students at four metro Institutes of Technology and Polytechnics: Otago Polytechnic, Waikato Institute of Technology, Wellington Institute of Technology, and Christchurch Polytechnic Institute of Technology. After the students had completed the theoretical maths course, they were tested to see if they could apply this maths knowledge to solve industry related problems. These students did not undergo a workshop to explain the industry related maths problem concept as detailed in Phase 1. Seventy six students from these four metro Institutes of Technology and Polytechnics registered in the maths class and 36 of these students completed the test for the research.

The findings of this project were:

1. Most first year BEngTech students who completed the test had difficulty with applying their maths knowledge to industry oriented problems.
2. The engineering industry's need for mathematical knowledge in the workplace varied between the different majors, i.e. mechanical, civil or electrical.

The recommendations of this project are:

1. That further research is carried out in order to determine whether students need to be taught additional skills for them to be able to apply theoretical knowledge in industry contexts. One example is the development of scenario or exemplar questions.
2. That the same 2014 cohort is tested in their final year (2016) with the industry applied assessment to see if their ability to apply their maths knowledge has been improved by their exposure to the entire curriculum, not just maths.
3. That the Stage 1 half day workshop is replicated for all first year maths students in 2015 across the four ITPs following which these students be tested with both the standard assessment and the industry applied assessment.
4. That a more detailed research project be formulated that includes interviews with employers to investigate their specific needs with regard to the application of

mathematical knowledge in the industry and continues to build a library of real world problems.

5. That the research extends to a Phase Three which would explore the implementation of industry project based learning as an alternative to the current method used of theoretical based teaching.

INTRODUCTION

The Bachelor of Engineering Technology (BEngTech) degree has been developed as a joint venture between industry and the six largest New Zealand Institutes of Technology and Polytechnics (ITPs): Unitec Institute of Technology (Unitec), Manukau Institute of Technology (MIT), Wellington Institute of Technology (WelTec), Christchurch Polytechnic Institute of Technology (CPIT), Waikato Institute of Technology (Wintec) and Otago Polytechnic (OP) who form the metropolitan group of ITPs (Metro Group). This degree is provisionally accredited by Institution of Professional Engineers New Zealand (IPENZ) and is recognised as meeting the initial academic requirements for Engineering Technologists, as defined in the Sydney Accord – an agreement developed for engineering technologists. Currently the programme has three majors: Civil, Electrical and Mechanical Engineering.

New Zealand has a high demand for an increased number of Engineering Technologists. As this new degree has an industry oriented commitment it is important that a new teaching and learning methodology is developed in conjunction with industry leaders. The mathematics component is the key to enhancing the capability of students to perform well in other engineering courses within the programme as it is central to engineering practice (Paas et al., 2004; Hawera & Taylor, 2007 and Cardella, 2010).

Engineering mathematics, which includes a branch of applied mathematics, is taught in the first year of the BEngTech as a compulsory course. Thereafter students are not required to enrol in a mathematics course again and have no chance to practise most of the teaching content learnt from Engineering Mathematics.

Mathematics is an integral part of engineering education and is currently being taught in a similar way to that of similar science-based degrees, i.e. as a pure theoretical subject. The student profile of BEngTech students is however one where students are seeking a professional or applied qualification rather than an academic pathway to further study. This leads to the possibility that the BEngTech students will understand maths concepts better if they are taught in an applied manner instead of as a purely theoretical subject.

This project contributes to the development of knowledge around industrial subject oriented teaching and learning strategies and as such responds to the current government desire for preparing students for work.

The Kaitohutohu at OP assisted with consultation for the project to gain ethical approval in 2013, and then extended ethical approval in 2014. The project has been granted ethical approval at the three other ITPs involved in this project: Wintec, WelTec, and CPIT.

PROJECT AIMS AND OBJECTIVE

This project aimed to explore whether an industry oriented mathematics teaching strategy would improve the mathematical achievement of learners in the Metro Group BEngTech programme. This is important to the learner as achievement in mathematics is integral to other aspects of the engineering programme. Understanding the mathematical underpinning of engineering processes is a basic building block of all engineering disciplines, although the ways these are applied to the different engineering specialties does differ. The research hypothesis was: *the needs of industry may require a shift from the teaching of generic or purely theoretical mathematics to the industry specific mathematics courses in Civil, Electrical and Mechanical Engineering.*

LITERATURE REVIEW

The mathematics component in a BEngTech programme is critical to enhancing the capability of students to perform well in other courses (Cardella, 2010; Gainsburg, 2006, and Engelbrecht et al., 2012). There is a plethora of research to support an industry oriented approach to teaching and learning mathematics in electronic and electrical engineering (Qi & Cannan, 2005, and Qi & Cannan, 2006b) and undergraduate software engineering courses (Alsmadi & Hanandeh, 2011, and Su et al., 2007) but there is currently a lack of literature with regard to civil and mechanical engineering.

In traditional technical teaching methodologies the conventional educational pathway is to build foundation learning through subject based teaching math, physics and science independently (Bachelor of Engineering, 2012; Bachelor of Engineering Degree structure, 2012, and Engineering technologist, 2012). Subjects based on the knowledge required for the discipline usually follow on from this. The problem with this traditional methodology of learning is that there is no close relationship with industry requirements. Students may well graduate with no industry oriented learning experience prior to their first job. Industry oriented methodology is learning from an industry perspective (Industry-oriented education, 2012, and Qi & Cannan, 2004).

As an example, the course of Electronics Technology in the BEngTech at Unitec Institute of Technology was directly linked to industry and the focus was on an industry oriented product such as a Switch-mode power supply (Qi & Cannan, 2005, and Qi & Cannan, 2006b). The focus for learning was product design, application and operation of electronic components and circuitry. Initially students received a demonstration and the product enclosure was opened to investigate inside. The internal components forming the topics for study included the mechanical design for the enclosure, electronic design including the PCB (Printed Circuit Board) and embedded software design. An industry oriented product was used to simulate industry conditions where students will gain invaluable insight into design technology, operational procedure and programming techniques (Qi & Cannan, 2005). All foundation skills can be taught within these studies and the students are well prepared to develop further knowledge and skills (Qi & Cannan, 2007a) in their final year through cooperative education with industry.

Under this model mathematics was totally integrated into the compulsory technical courses rather than as a standalone course.

This approach can be applied to a traditional engineering undergraduate programme (Qi, 2008b), an industry oriented and multi-discipline undergraduate degree (Qi & Cannan, 2007b) and post-graduate programmes (Qi, 2008a, and Qi, 2009). For example, a “bridging” technology course was designed to enable a Bachelor of Design graduate in Unitec to enter a Master of Design programme in Unitec (Qi, 2008a, and Qi, 2009). This teaching approach requires a change in role for the lecturer. In industry orientated education the lecturers needed to build their industry background. The academic staff were encouraged to join the student industry projects as supervisors to improve their industry background, while industry staff were invited to teach as guest or part-time lecturers (Qi & Cannan, 2007c).

The researchers were aware that for some learners (Māori), mathematics and how it is offered outside of a context is viewed by some as a subject that promotes the values of the dominant culture (Hawera & Taylor, 2007). The Māori concept of ‘Ako’ encompasses learning and teaching as a process intertwined with concepts of mōhio (knowledge) and māramataka (understanding) which ultimately results in mātauraka (wisdom). This synthesises people, ideas and the environment as part of a greater whole; “Education is considered to be a holistic enterprise, so mathematics should be integrated with everyday life” (Hawera & Taylor, 2007). Hawera & Taylor’s (2007) study of Māori school students found that students had difficulty applying maths to everyday life, i.e. placing maths learning within a context that has meaning for the learner/s. This would suggest that a more applied approach to maths teaching would also benefit Māori learners.

METHODOLOGY

The research was undertaken as action research in two phases during 2013 (phase 1) and 2014 (phase 2). The first phase was undertaken in 2013 by the project team from OP. An industry survey (20 requests were sent to engineering managers and six of them were completed and returned) was conducted to determine the aspects of the current maths course most needed in industry. The full questionnaire can be found in Appendix 1. Further interaction with industry groups obtained authentic and appropriate case studies to use as teaching examples. Individual interviews were undertaken with the principal lecturers in the three engineering majors, Electrical, Civil and Mechanical, within the BEngTech programme at OP. Some industry examples of maths application from the three majors were selected during discussions with the researchers and industry people. These samples were used in a student workshop in addition to the standard curriculum from previous years. The students were assessed in the same way as the previous year cohorts, using theoretical based questions. A comparison was also made between the 2012 and 2013 exam results and pass rates at OP. These students gave informed consent and qualitative feedback.

In phase 2 the method was amended with the involvement of the three other metro ITPs: Wintec, Weltec, and CPIT. Initially it was planned to gather additional data from other metro ITPs (such

as assignment results of the previous two years cohorts) to identify the pass rate and the grade status of students in each major, as well as to determine their base level of understanding to use as comparative data with phase 1 research. However, following discussion, this was changed to provide a formative test in the existing 2014 teaching plan for the maths course within the Metro Group BEng Tech programme. This was additional, without modifying any teaching content and learning outcomes in the class.

The formative test was developed in two parts: The first question was generic and involved finding the determinant and inverse of a matrix related to a system of simultaneous equations. The second question allowed the students to choose from a) civil, b) electrical, or c) mechanical areas but all involved setting up and solving a system of simultaneous equations using a matrix method. The results of the first question could then be compared with the second. Appendix 4 details the formative test that was trialed on the students at the four Metro ITPs.

RESULTS AND FINDINGS

Summary of findings

The study produced four broad findings.

Phase 1: The OP team found:

The engineering industry requirement for mathematical knowledge in the workplace varied between the different majors, i.e. mechanical, civil or electrical.

Phase 2: The collaborative project found:

Students from all the current engineering math classes in the research programme had difficulty applying their knowledge to solve an industry application, even if their mathematical knowledge was rated satisfactory in the theoretical application of the test.

These broad findings are discussed in more detail below.

Analysis of Phase1 results

Survey of engineering industry leaders

Phase 1 surveyed engineering industry leaders using the full questionnaire shown in Appendix 1. As shown in Appendix 5, it was found that while the BEng Tech taught a generic maths course, depending on the industry major, the engineering industry had differing needs for maths knowledge.

Teaching using industry samples

After some industry examples of maths application from the three majors (shown in Appendices 2 and 3) were delivered in a workshop in addition to the standard curriculum from previous

years, the students were assessed in the same way as the previous year cohorts, using theoretical based questions. A comparison was made between the 2012 and 2013 exam results and pass rates at OP. The detailed analysis of the student results in the standard theoretical tests is shown in Appendix 3. OP had civil and mechanical students for this research.

Our conjecture was that the civil major students in 2013 with the benefit of the workshop would do better than the 2012 students on the Matrices question while the mechanical major students in 2013 would do better than the 2012 students on the Series question.

Base mean

The means for each question was calculated for the different years and streams. As shown in Figures 1 and 2, there is a very small change between the marks obtained in 2012 and 2013 for each stream. However they are the opposite of what was conjectured. From looking at the graphs we can see that:

- Civil stream students dropped 3% on matrices question
- Mechanical stream students gained 1% on matrices question
- Civil stream students gained 2% on series question
- Mechanical stream students dropped 1% on series question.

These are very small changes in percentage so no immediate significance is seen.

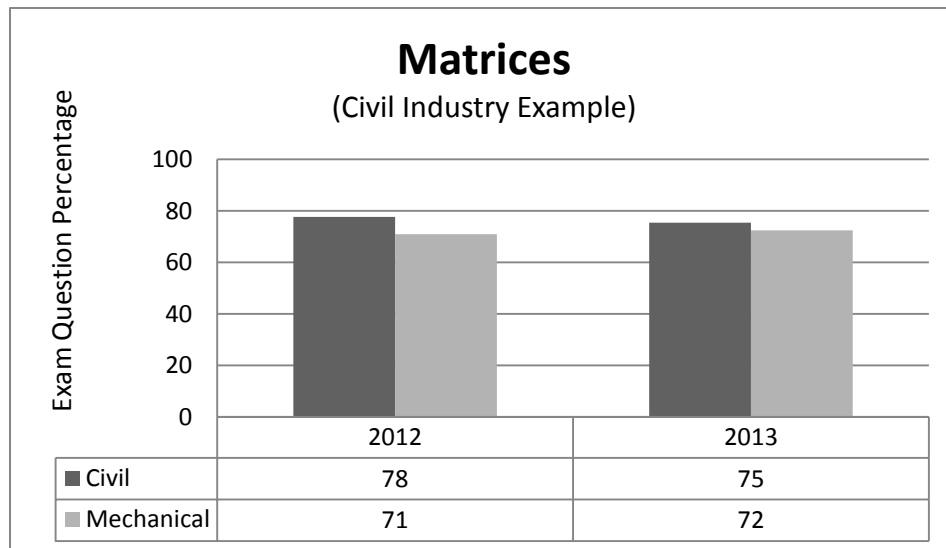


Figure 1 Civil engineering sample

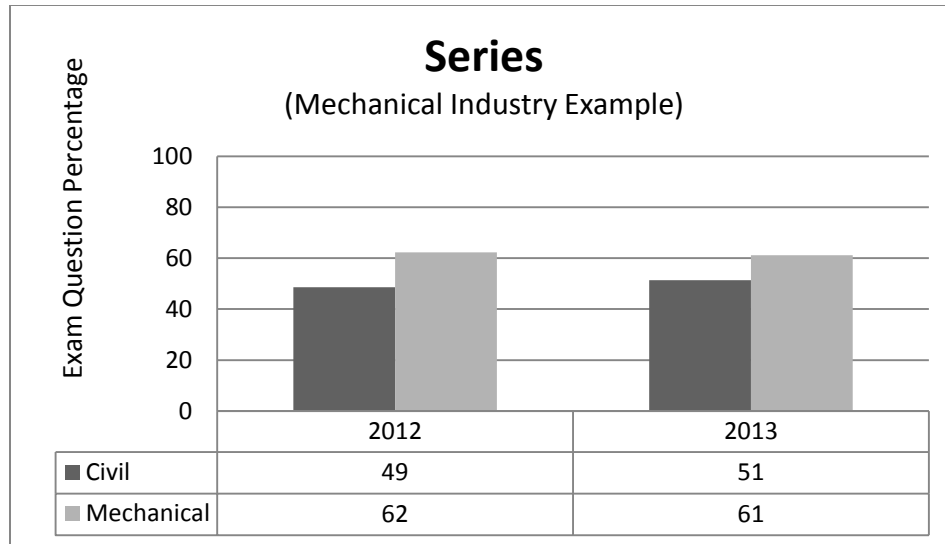


Figure 2 Mechanical engineering sample

Statistical testing

A test was run with a 95% significance level to examine the change in the Phase 1 data. The same test was run on each of the questions for both streams to check for any significance across the board.

F-TEST

An F-test is used first to determine the variance of the data collected as showed in Table 3-6.

Table 1 F-test at Civil Stream (Matrices)

Civil Stream (Matrices)		
	2012	2013
Mean	77.5568182	75.37879
Variance	440.433146	339.7039
Observations	8	6

Table 2 F-test at Civil Stream (Series)

Civil Stream (Series)		
	2012	2013
Mean	48.61111	51.38889
Variance	359.3474	869.5988
Observations	8	6

Table 3 F-test at Mechanical Stream (Matrices)

Mechanical Stream (Matrices)		
	2012	2013
Mean	70.9090909	72.34848
Variance	813.59045	673.2094
Observations	10	12

Table 4 F-test at Mechanical Stream (Matrices)

Mechanical Stream (Series)		
	2012	2013
Mean	62.22222	61.11111
Variance	375.8573	735.1291
Observations	10	12

The variances are all greatly different for each data set. This determines that any t-tests used will need to be performed with unequal variances in mind.

T-test

A t-test was performed on each selection of data to determine if it is of any statistical importance.

Null hypothesis: that the difference in mean between 2013 and 2012 is equal.

Alternative hypothesis: that the difference in mean for 2013 is greater than in 2012.

Significance level: 95%

A t-test was performed to check if there was any statistical evidence that the marks in 2013 were significantly greater when compared with the 2012 marks. A two tailed test was performed to check if there is any statistical evidence that the marks in 2013 were significantly different when compared with the 2012 marks.

Table 5 T-test at Civil Stream (Matrices)

Civil Stream (Matrices)	
t-test: Two-sample Assuming Unequal Variances	
Hypothesized Mean Difference	0
df	12
t Stat	0.20610704
P(T<=t) one-tail	0.42008135
t Critical one-tail	1.78228756

P(T<=t) two-tail	0.8401627
t Critical two-tail	2.17881283

Table 6 T-test at Civil Stream (Series)

Civil Stream (Series)	
t-test: Two-sample assuming Unequal Variances	
Hypothesized Mean Difference	0
df	8
t Stat	-0.2015999
P(T<=t) one-tail	0.42263064
t Critical one-tail	1.85954804
P(T<=t) two-tail	0.84526127
t Critical two-tail	2.30600414

Table 7 T-test at Mechanical Stream (Matrices)

Mechanical Stream (Matrices)	
t-test: Two-Sample Assuming Unequal Variances	
Hypothesized Mean Difference	0
df	18
t Stat	-0.1227699
P(T<=t) one-tail	0.45182486
t Critical one-tail	1.73406361
P(T<=t) two-tail	0.90364972
t Critical two-tail	2.10092204

Table 8 T-test at Mechanical Stream (Series)

Mechanical Stream (Series)	
t-test: Two-sample Assuming Unequal Variances	
Hypothesized Mean Difference	0
df	20
t Stat	0.11175755
P(T<=t) one-tail	0.45606483
t Critical one-tail	1.72471824
P(T<=t) two-tail	0.91212967
t Critical two-tail	2.08596345

In Tables 7 to 10 we can clearly see that the Critical t value is much higher than the t stat. This means that there is no statistical importance between the marks from 2012 and 2013 in the algebra exam regarding the matrices and series questions.

Our conjecture was that the civil major students in 2013 with the benefit of the workshop would do better than the 2012 students on the Matrices question while the mechanical major students in 2013 would do better than the 2012 students on the Series question.

The t-test results show that the conjecture has no statistical evidence to back it up, and we must instead accept the null hypothesis stated, namely that the difference in mean between 2013 and 2012 is equal. Below is a graph showing the overall exam mean for the combined streams in both years: It is interesting to note that the difference in the overall means for both years' exams results is only 0.2%. This shows great consistency and is a good indicator that we could expect the whole exam to have been consistent with no major change in exam marks as showed in Figure 3.

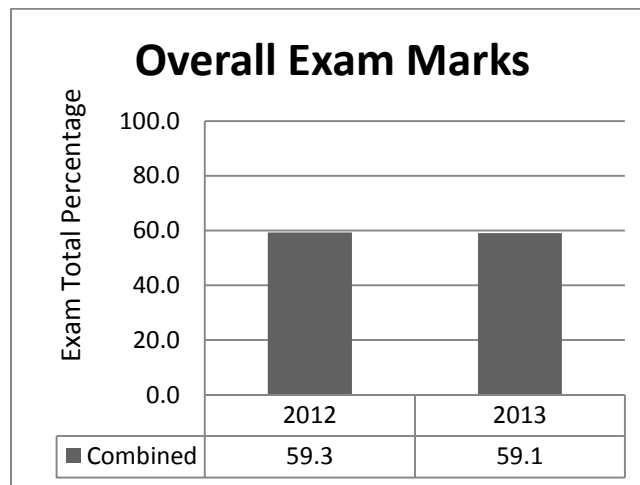


Figure 3

Analysis of Phase 2 results

Students were offered two questions. Question 1 (Q1) was a standard findings theoretical maths question and question 2 (Q2) asked students to apply their theoretical knowledge to an industry-based question. The results are separated and analysed by student major of mechanical, civil, or electrical engineering across the four ITPs. The results were compared after all tests were completed so there is no way of matching the test results to specific students, and any results of the study will only report the anonymized data.

The collated data from the four participating ITPs is shown in Table 11. Sixty one per cent of students completed both Q1 and Q2.

1. Students did a lot better on Q1 than Q2 scoring an average of 70% on Q1 and 35% on Q2 and the difference is statistically significant ($p < 0.00000001$).

2. If we only consider those students who attempted Q2, it can still be seen that they did better on Q1 than Q2 although the difference is less marked. Their average scores were 79% for Q1 and 59% for Q2 and once again the difference is statistically significant ($p < 0.001$).

Table 9 Collated Data from the four institutes

Student category	Number	Q1 mean score (%)	Q2 mean score (%)
All	76	70	35
Attempted BOTH questions	46	79	59
Attempted Q1 only	30	56	
Mechanical attempted both questions	18	73	61
Electrical attempted both questions	8	83	67
Civil attempted both questions	20	82	54

3. Table 12 shows the correlation coefficients of the comparison. There is little correlation between Q1 and Q2 for any of the groups of students as the correlation coefficients are Mechanical 0.48, Electrical 0.22, and Civil 0.13. This poor correlation can be seen graphically in Figures 4 to 6.

Table 10 Correlation coefficient, r, between Q1 and Q2

	r (Q1 and Q2)
Mechanical	0.48
Electrical	0.22
Civil	0.13

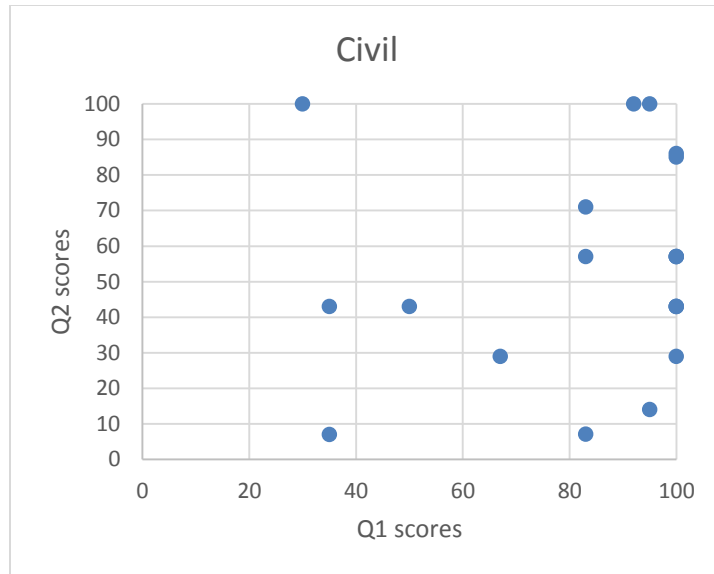


Figure 4 Student scores - Civil Q2 vs Q1

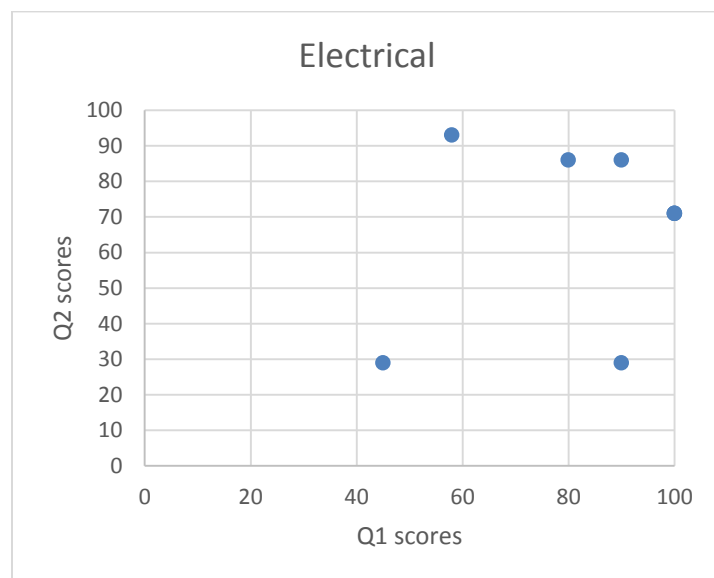


Figure 5 Student scores - Electrical Q2 vs Q1

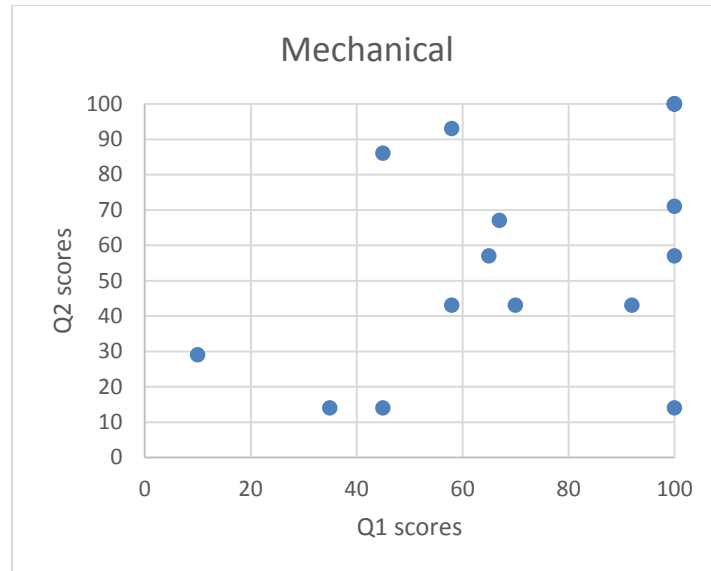


Figure 6 Student scores - Mechanical Q2 vs Q1

4. Many students did not appear to have enough confidence to apply skills shown in Q1 to the industry-oriented application in Q2.
5. About half the students used the inverse matrix method for Q1 part (d), and about half used the Gaussian method, although parts (a) to (c) guided them towards the inverse matrix method. A few students only used their calculator, although full working had been requested. In Q2, most students used the Gaussian method. This method appears to be the most popular of the three matrix methods taught in the course.

Nature of the industry oriented questions

For industry oriented questions (Q2), students needed skills that were additional to those needed for the theoretical questions (Q1) where the equations were given in standard textbook form and early parts of the question guided students towards a particular method. In Q2 however, students needed to rewrite the equations into standard textbook form and received no guidance towards a method. Thus they needed to read, understand, and interpret both words and diagrams.

In addition, the three options for Q2 had other characteristics that made them more difficult to analyse than Q1. These are shown in Table 13.

Table 11 Characteristics of Q2

Q2a	Needed to show where the equations came from
Civil	Needed to deal with zero coefficients One equation needed rearranging
Q2b	Needed to substitute values
Electrical	Needed to know Kirchoff's Laws
Q2c	There were four unknowns
Mechanical	Needed to deal with zero coefficients Needed to substitute values and these were not given until part (ii) of the question

Discussions with mathematics lecturers

As part of the action research process we discussed with mathematics lecturers whether they would expect to teach mathematics and whether their students would be taught how to apply this to industry applications in their engineering courses. We found that

- There are a number of additional skills needed to “use mathematics” and it appears that these skills, some of which are identified in these discussions, may not be being taught in either the mathematics or the engineering courses at year 1 of BEngTech.
- The mathematics lecturers expressed a willingness to teach these industry based problems. However, although they have background of basic engineering, they felt they are not necessarily confident or competent to teach engineering mathematical application. Similarly, engineering teachers understand the mathematics they use but may not feel confident about teaching that aspect.
- No evidence is available yet to show whether students would develop some of the additional skills needed to use maths as they continue in years 2 or 3 of their BEngTech programme.

CONCLUSION AND RECOMMENDATIONS

At phase 1, the engineering industry showed support for industry oriented teaching and learning. Their input identified however that there were some different needs for maths knowledge and variations in application between the different majors.

At phase 2, students in maths classes lack the skills to apply their mathematical knowledge to industry oriented questions. However, as the students tested were in their first year of study, there may be other variables that improve competence as students move through their entire course of study.

This raises further questions such as, what role does industry oriented teaching and learning have on a student's ability to apply mathematical knowledge? Would their achievement improve once they are taught industry based courses? We also noted whether traditional mathematics lecturers would expect to teach the mathematics and assume that students would be taught how to apply this to industry applications in their engineering courses. More research is needed to determine whether the additional skills needed by students to apply maths is developed as students progress in years 2 and 3 and whether it needs to be taught in either the mathematics or the engineering courses.

Based on reflections on this project, it is recommended that this project be extended to phase 3, which could address the following:

- Test the same 2014 cohort of students in their final year (2016) with the industry applied assessment to see if their ability to apply their maths knowledge has been improved by their exposure to the entire curriculum, not just maths.
- Replicate the phase 1 half day workshop for all first year maths students in 2015 across the four study ITPs and test these students with both the standard assessment and the industry applied assessment.
- Undertake more detailed research with employers about their specific needs with regard to the application of mathematical knowledge and continue to build a library of real world problems.
- Taking into account the applied nature of the BEngTech training this project could be extended to phase 3 to trial the implementation of project based learning as an alternative to the current method used of theoretical based teaching.

REFERENCES

- Alsmadi, I., Hanandeh, F. (2011). A framework for teaching software engineering introductory courses. *International Journal of Education Economics and Development*. 2: 4(380-397).
- Bachelor of Engineering. (2012). In Wikipedia, The Free Encyclopedia. Retrieved September 4, 2012, from http://en.wikipedia.org/w/index.php?title=Bachelor_of_Engineering&oldid=509733875
- Bachelor of Engineering Degree structure. (2012). In Canterbury University. Retrieved September 4, 2012, from http://www.canterbury.ac.nz/courses/undergrad/behons.shtml#unique_6
- Bachelor of Engineering Technology. (2012). In Otago Polytechnic. Retrieved September 4, 2012, from <http://www.otagopolytechnic.ac.nz/programmes/bachelor-of-engineering-technology.html>
- Cardella, M. E. (2010). Mathematical modelling in engineering design projects. In R. Lesh, P. Galbraith, C. Haines, & A. Hurford (Eds.), *Modeling students' mathematical modeling competencies* (Proc. ICTMA 13), (pp. 87-98). New York, NY: Springer.
- Coghlan, D. & Brannick, T. (2001). *Doing Action Research in Your Own Organisation*. London: Sage Publications, Ltd.
- Engelbrecht, J., Bergsten, C. & Kågesten, O. (2012). Conceptual and Procedural Approaches to Mathematics in the Engineering Curriculum: Student Conceptions and Performance. *Journal of Engineering Education* 101. 1 (Jan 2012): 138-162.
- Engineering technologist. (2012). In Wikipedia, The Free Encyclopedia. Retrieved September 4, 2012, from http://en.wikipedia.org/w/index.php?title=Engineering_technologist&oldid=509981246
- Gainsburg, J. (2006). The mathematical modeling of structural engineers. *Mathematical Thinking and Learning*, 8(1), 3-36.
- Hawera, N. & Taylor, M. (2007). "Māori and Mathematics: "Nā te mea he pai mō tō roro! (Because it's good for your brain!)" Findings from the New Zealand Numeracy Development Project 2007, http://www2.nzmaths.co.nz/Numeracy/References/comp07/tpt07_hawera_taylor.pdf
- Industry-oriented education. (2011, November 14). In Wikipedia, The Free Encyclopedia. Retrieved 01:47, September 4, 2012, from http://en.wikipedia.org/w/index.php?title=Industry-oriented_education&oldid=460607286
- Paas, F., Renkel, A. & Sweller, J. (2004). "Cognitive Load Theory: Instructional Implications of the Interaction between Information Structures and Cognitive Architecture". *Instructional Science* 32: 1–8.

- Qi, Z. & Cannan, J. (2004). *Industrial oriented teaching and learning strategies*, 5th Asia Pacific Cooperative Education Conference. Auckland, New Zealand, 1 December-3 December, Auckland, New Zealand.
- Qi, Z. & Cannan, J. (2005). Engineering Mathematics teaching methodology in Bachelor of Applied Technology (Electro-technology). In *I. Electronics New Zealand Conference (Ed.), 12th Electronics New Zealand Conference (pp. 225-230)*. Auckland: Electronics New Zealand Conference, Inc. Paper presented at the Department of Electrical & Computer Engineering, Manukau Institute of Technology, 14 November-15 November, Auckland.
- Qi, Z. & Cannan, J. (2006a). Student/Industry Project in Industrial Oriented Undergraduate Degree in Electro-technology. In R. K. Coll (Ed.), *Conference Proceedings: New Zealand Association for Cooperative Education Annual Conference*. Queenstown: NZACE. Paper presented at the NZACE, 27 April-28 April, Queenstown.
- Qi, Z. & Cannan, J. (2006b). *Engineering Mathematics in Industrial Orientated Teaching and Learning*, Proceedings of WACE Asia-Pacific Regional Conference 2006. Shanghai: WACE Asia-Pacific Regional Conference, 26 June-28 June, Shanghai.
- Qi, Z. & Cannan, J. (2007a) *Industrial Oriented Project based learning: Exploring a model for real world learning from the laboratory to the workplace*, in The 15th World Conference on Cooperative Education, 26 June-29 June, Singapore.
- Qi, Z. & Cannan, J. (2007b) *An Industrial Oriented and multi-discipline undergraduate degree*, in The 15th World Conference on Cooperative Education, 26 June-29 June, Singapore.
- Qi, Z. & Cannan, J. (2007c). *The Changing role of the Lecturer in Industry Orientated Education*, Tenth Annual NZACE Conference. Rotorua, 19 April-20 April, Rotorua, New Zealand.
- Qi, Z. (2008a). Work in Progress: *A Master of Design Program Collaborating with Electronic Engineering and Technology*, the 38th Annual Frontiers in Education (FIE) Conference, 22 October-25 October, New York, USA.
- Qi, Z. (2008b). *Industry Oriented Teaching and Learning Strategies Applied to Traditional Engineering Undergraduate Program*, in the 38th Annual Frontiers in Education (FIE) Conference, 22 October-25 October, New York, USA.
- Qi, Z. (2009) *Work in Progress: A Course Design for a Master of Design Program Linked to the Electronic and Computer Engineering Graduates*, the 39th Annual Frontiers in Education (FIE) Conference, 18 October-21 October, San Antonio, Texas, USA.
- Su, H., Jodis, S., and Zhang, H. (2007). Providing an integrated software development environment for undergraduate software engineering courses. , 23, (143-149).

Appendix 1: Question list for focus group with industry leaders

Question list for focus group with industry leaders

The purpose of this research is to:

- identify math concepts used in your field

For any topic below which you would apply “our engineers must know”, could you please provide an example of where this concept is used in your field.

Tick below box to identify your specialist area

<input type="checkbox"/> Civil Engineering
<input type="checkbox"/> Electrical Engineering
<input type="checkbox"/> Mechanical Engineering

ALGEBRA

1 FUNCTIONS AND COORDINATE SYSTEMS

1.1 Define and graph a relation and its inverse where the relation is:

- 1.1.1 simple polynomial eg. $y = kx^2$
- 1.1.2 exponential
- 1.1.3 circular
- 1.1.4 hyperbolic

<input type="checkbox"/> Our engineers must know
<input type="checkbox"/> Our engineers needn't know
<input type="checkbox"/> not sure

1.2 Convert among common coordinate systems

<input type="checkbox"/> Our engineers must know
<input type="checkbox"/> Our engineers needn't know
<input type="checkbox"/> not sure

1.3 Graph plane curves in polar co-ordinates

<input type="checkbox"/> Our engineers must know
<input type="checkbox"/> Our engineers needn't know
<input type="checkbox"/> not sure

2. VECTOR ALGEBRA

2.1 Describe a 3 space vector as an ordered triple and in terms of the unit vectors \mathbf{i} , \mathbf{j} , \mathbf{k} (include discussion of 2 space)

<input type="checkbox"/> Our engineers must know
<input type="checkbox"/> Our engineers needn't know
<input type="checkbox"/> not sure

2.2 Perform vector addition, subtraction and multiplication by scalar quantities

<input type="checkbox"/> Our engineers must know
<input type="checkbox"/> Our engineers needn't know
<input type="checkbox"/> not sure

2.3 Calculate the magnitude and the directed unit vector corresponding to a vector \mathbf{v}

<input type="checkbox"/> Our engineers must know
<input type="checkbox"/> Our engineers needn't know
<input type="checkbox"/> not sure

2.4 Find the vector normal and vector tangent to a simple curve at a specified point

Find: (a) the vector between 2 points A, B in 3 space
(b) the distance between the points A,B

<input type="checkbox"/> Our engineers must know
<input type="checkbox"/> Our engineers needn't know
<input type="checkbox"/> not sure

2.5 Define the scalar product and describe its geometric significance in terms of projections

<input type="checkbox"/> Our engineers must know
<input type="checkbox"/> Our engineers needn't know
<input type="checkbox"/> not sure

2.6 Apply the scalar product to simple physical problems

<input type="checkbox"/> Our engineers must know
<input type="checkbox"/> Our engineers needn't know
<input type="checkbox"/> not sure

2.7 Define and find direction cosines

<input type="checkbox"/> Our engineers must know
<input type="checkbox"/> Our engineers needn't know
<input type="checkbox"/> not sure

2.8 Define and evaluate determinants (up to order 3)

<input type="checkbox"/> Our engineers must know
<input type="checkbox"/> Our engineers needn't know
<input type="checkbox"/> not sure

2.9 Define the cross product and describe its geometric properties

<input type="checkbox"/> Our engineers must know
<input type="checkbox"/> Our engineers needn't know
<input type="checkbox"/> not sure

2.10 Verify the distributive law for the cross product

<input type="checkbox"/> Our engineers must know
<input type="checkbox"/> Our engineers needn't know
<input type="checkbox"/> not sure

- 2.11 Apply the cross product to simple physical problems
- | |
|---|
| <input type="checkbox"/> Our engineers must know |
| <input type="checkbox"/> Our engineers needn't know |
| <input type="checkbox"/> not sure |
- 2.13 State the equation of a plane
- | |
|---|
| <input type="checkbox"/> Our engineers must know |
| <input type="checkbox"/> Our engineers needn't know |
| <input type="checkbox"/> not sure |
- 2.14 Find a vector $\underline{\mathbf{N}}$, normal to a plane
- | |
|---|
| <input type="checkbox"/> Our engineers must know |
| <input type="checkbox"/> Our engineers needn't know |
| <input type="checkbox"/> not sure |
- 2.15 Describe the algebraic and geometric properties of the triple scalar product
- | |
|---|
| <input type="checkbox"/> Our engineers must know |
| <input type="checkbox"/> Our engineers needn't know |
| <input type="checkbox"/> not sure |
- 2.16 Apply vectors to engineering applications
- | |
|---|
| <input type="checkbox"/> Our engineers must know |
| <input type="checkbox"/> Our engineers needn't know |
| <input type="checkbox"/> not sure |
- 3 LINEAR ALGEBRA
- 3.1 Apply the rules for matrix addition, subtraction and scalar multiplication
- | |
|---|
| <input type="checkbox"/> Our engineers must know |
| <input type="checkbox"/> Our engineers needn't know |
| <input type="checkbox"/> not sure |
- 3.2 Apply the rule for matrix multiplication
- | |
|---|
| <input type="checkbox"/> Our engineers must know |
| <input type="checkbox"/> Our engineers needn't know |
| <input type="checkbox"/> not sure |
- 3.3 State the additive and multiplicative matrix identities
- | |
|---|
| <input type="checkbox"/> Our engineers must know |
| <input type="checkbox"/> Our engineers needn't know |
| <input type="checkbox"/> not sure |
- 3.4 Investigate the commutative, associative and distributive properties of matrices
- | |
|---|
| <input type="checkbox"/> Our engineers must know |
| <input type="checkbox"/> Our engineers needn't know |
| <input type="checkbox"/> not sure |
- 3.5 Apply the rules governing elementary row operations to obtain an inverse matrix
- | |
|--|
| <input type="checkbox"/> Our engineers must know |
|--|

3.6 Show that a consistent set of linear equations may be represented in matrix form and hence find a solution set (using a variety of standard methods)

<input type="checkbox"/> Our engineers needn't know
<input type="checkbox"/> not sure

3.7 Understand the term 'row echelon form'

<input type="checkbox"/> Our engineers must know
<input type="checkbox"/> Our engineers needn't know
<input type="checkbox"/> not sure

3.8 Find an inverse matrix by cofactors

<input type="checkbox"/> Our engineers must know
<input type="checkbox"/> Our engineers needn't know
<input type="checkbox"/> not sure

3.9 Determine the Eigen values of a matrix

<input type="checkbox"/> Our engineers must know
<input type="checkbox"/> Our engineers needn't know
<input type="checkbox"/> not sure

3.10 Determine the Eigen vectors for a matrix

<input type="checkbox"/> Our engineers must know
<input type="checkbox"/> Our engineers needn't know
<input type="checkbox"/> not sure

3.11 Apply Eigen vectors to the linear transformation of Cartesian coordinate systems

<input type="checkbox"/> Our engineers must know
<input type="checkbox"/> Our engineers needn't know
<input type="checkbox"/> not sure

3.12 Use matrices to solve engineering problems

<input type="checkbox"/> Our engineers must know
<input type="checkbox"/> Our engineers needn't know
<input type="checkbox"/> not sure

4 COMPLEX ALGEBRA

4.1 Convert between Cartesian and polar forms

<input type="checkbox"/> Our engineers must know
<input type="checkbox"/> Our engineers needn't know

not sure

4.2 Represent these forms on a drawing of the complex plane

Our engineers must know
 Our engineers needn't know
 not sure

4.3 Apply Euler's formula

Our engineers must know
 Our engineers needn't know
 not sure

4.4 Derive the fundamental identity $e^{j\pi} + 1 = 0$

Our engineers must know
 Our engineers needn't know
 not sure

4.5 Verify the complex links between $\sin z$, $\cos z$, $\sinh z$ and $\cosh z$

Our engineers must know
 Our engineers needn't know
 not sure

4.6 Deduce and use Osborne's Rule

Our engineers must know
 Our engineers needn't know
 not sure

4.7 Define and evaluate the following functions: $\sin z$, $\cos z$, $\ln z$, $\sinh z$, $\cosh z$, z^n and combinations of these as appropriate

Our engineers must know
 Our engineers needn't know
 not sure

4.8 Find roots of complex numbers

Our engineers must know
 Our engineers needn't know
 not sure

4.9 Show the importance of complex numbers in professional engineering calculations

<input type="checkbox"/> Our engineers must know
<input type="checkbox"/> Our engineers needn't know
<input type="checkbox"/> not sure

5 SERIES

5.1 Understand the term "limit" and the notion of convergence

<input type="checkbox"/> Our engineers must know
<input type="checkbox"/> Our engineers needn't know
<input type="checkbox"/> not sure

5.2 Use L'Hopital's Rule

<input type="checkbox"/> Our engineers must know
<input type="checkbox"/> Our engineers needn't know
<input type="checkbox"/> not sure

5.3 Write a Maclaurin Series for transcendental functions

<input type="checkbox"/> Our engineers must know
<input type="checkbox"/> Our engineers needn't know
<input type="checkbox"/> not sure

CALCULUS

1 DIFFERENTIAL CALCULUS

1.1 Review of differentiation concepts and methods

<input type="checkbox"/> Our engineers must know
<input type="checkbox"/> Our engineers needn't know
<input type="checkbox"/> not sure

1.2 Perform implicit differentiation

<input type="checkbox"/> Our engineers must know
<input type="checkbox"/> Our engineers needn't know
<input type="checkbox"/> not sure

1.3 Recognise a composite function and state the Chain Rule for its derivative

<input type="checkbox"/> Our engineers must know
<input type="checkbox"/> Our engineers needn't know
<input type="checkbox"/> not sure

1.4 Understand the terms concavity, critical point, inflexion, continuity, increasing and decreasing

<input type="checkbox"/> Our engineers must know
--

1.5	Define and apply the ∂ operator	<input type="checkbox"/> Our engineers needn't know <input type="checkbox"/> not sure
1.6	Apply the Chain Rule for functions of more than one variable	<input type="checkbox"/> Our engineers must know <input type="checkbox"/> Our engineers needn't know <input type="checkbox"/> not sure
1.7	Use partial derivatives to explore 1.4	<input type="checkbox"/> Our engineers must know <input type="checkbox"/> Our engineers needn't know <input type="checkbox"/> not sure
1.8	Apply partial differentiation to solve practical engineering problems	<input type="checkbox"/> Our engineers must know <input type="checkbox"/> Our engineers needn't know <input type="checkbox"/> not sure
2	INTEGRAL CALCULUS	
2.1	Review of integration concepts and methods	<input type="checkbox"/> Our engineers must know <input type="checkbox"/> Our engineers needn't know <input type="checkbox"/> not sure
2.2	Find an integral by expansion to partial fractions	<input type="checkbox"/> Our engineers must know <input type="checkbox"/> Our engineers needn't know <input type="checkbox"/> not sure
2.3	Find an integral by trigonometric or hyperbolic substitution	<input type="checkbox"/> Our engineers must know <input type="checkbox"/> Our engineers needn't know <input type="checkbox"/> not sure
2.4	Apply integration techniques to find the length of a plane curve	<input type="checkbox"/> Our engineers must know <input type="checkbox"/> Our engineers needn't know <input type="checkbox"/> not sure
2.5	Apply integration techniques to find the area of a surface of revolution	<input type="checkbox"/> Our engineers must know <input type="checkbox"/> Our engineers needn't know <input type="checkbox"/> not sure
2.6	Apply integration techniques to find the volume of a solid of revolution	<input type="checkbox"/> Our engineers must know <input type="checkbox"/> Our engineers needn't know

- 2.7 Apply the method of integration by parts to integrate products of functions (excluding reduction formulae) not sure
- Our engineers must know
 Our engineers needn't know
 not sure
- 2.8 Evaluate and use double integrals
- Our engineers must know
 Our engineers needn't know
 not sure
- 2.9 Apply integration to solve practical engineering problems
- Our engineers must know
 Our engineers needn't know
 not sure

3 DIFFERENTIAL EQUATIONS

- 3.1 Define the terms:
- 3.1.1 differential equation
- 3.1.2 order
- 3.1.3 degree
- 3.1.4 initial condition
- 3.1.5 boundary condition
- 3.1.6 general solution

Our engineers must know
 Our engineers needn't know
 not sure

- 3.2 Distinguish first and second order Linear D.Es

Our engineers must know
 Our engineers needn't know
 not sure

- 3.3 Recognise that the general solution of a D.E. describes a family of curves and that the particular solution describes a unique curve

Our engineers must know
 Our engineers needn't know
 not sure

- 3.4 Solve a first order linear DE by:
- 3.4.1 direct integration

- 3.4.2 separation of variables
- 3.4.3 integrating factor
- 3.4.4 substituting $y = bx$ for homogeneous DEs

<input type="checkbox"/> Our engineers must know
<input type="checkbox"/> Our engineers needn't know
<input type="checkbox"/> not sure

3.5 Solve a homogeneous second order D.E. of the form
 $ay'' + by' + cy = 0$

for all real a, b, c

<input type="checkbox"/> Our engineers must know
<input type="checkbox"/> Our engineers needn't know
<input type="checkbox"/> not sure

3.6 Apply the above techniques, where appropriate, to find the particular solution for engineering problems.

<input type="checkbox"/> Our engineers must know
<input type="checkbox"/> Our engineers needn't know
<input type="checkbox"/> not sure

4 NUMERICAL METHODS

4.1 Construct Newton/Gregory difference tables

<input type="checkbox"/> Our engineers must know
<input type="checkbox"/> Our engineers needn't know
<input type="checkbox"/> not sure

4.2 Find an interpolating polynomial from data

<input type="checkbox"/> Our engineers must know
<input type="checkbox"/> Our engineers needn't know
<input type="checkbox"/> not sure

4.3 Use an interpolating polynomial to determine the value of slope at a particular point.

<input type="checkbox"/> Our engineers must know
<input type="checkbox"/> Our engineers needn't know
<input type="checkbox"/> not sure

4.4 Use the trapezium rule.

<input type="checkbox"/> Our engineers must know
<input type="checkbox"/> Our engineers needn't know

<input type="checkbox"/> not sure

4.5 Use Simpson's rule.

<input type="checkbox"/> Our engineers must know
<input type="checkbox"/> Our engineers needn't know
<input type="checkbox"/> not sure

4.6 Apply numerical techniques to solve problems in Calculus

<input type="checkbox"/> Our engineers must know
<input type="checkbox"/> Our engineers needn't know
<input type="checkbox"/> not sure

4.7 Use an iterative method (such as Euler's method) to solve simple first order differential equations.

<input type="checkbox"/> Our engineers must know
<input type="checkbox"/> Our engineers needn't know
<input type="checkbox"/> not sure

4.8 Know that a numerical process has an associated error bound and that such a bound should be evaluated and along presented with the numerical answer itself

<input type="checkbox"/> Our engineers must know
<input type="checkbox"/> Our engineers needn't know
<input type="checkbox"/> not sure

Appendix 2: Industry sample for civil engineering

The following sample was created based on the response from engineers in the relevant specialist “must know” as shown in Appendix 1.

CIVIL

1.1 Define and graph a relation and its inverse where the relation is:

Civil engineers would plot several parameters to reach substantial conclusions e.g., plot stress-strain curves to get the dependable yield strength of steel, time and spectral acceleration to get the design spectra etc.

Example 1: Readings from a typical steel test, in terms of force and displacement are given below. You are required to draw a typical stress strain plot and find the yield strength (i.e., at the first change of slope), elastic modulus (i.e., the slope of initial straight line) and ultimate strength of the reinforcement steel (i.e., maximum stress before breakage).

[Note: Cross-sectional area of the bar (12 mm diameter bar) = 113.1 mm² and Gauge length = 100 mm]

Sr. No.	Force (kN)	Displacement (mm)
1	0.00	0.00
2	32.78	0.15
3	47.92	0.22
4	58.67	0.28
5	63.71	0.32
6	66.80	0.35
7	68.57	0.38
8	69.96	0.40
9	70.94	0.42
10	71.75	0.44
11	74.75	0.54
12	75.97	0.67
13	77.17	1.02
14	76.63	1.45
15	75.02	1.83
16	68.33	2.28

1.1.1 simple polynomial eg. $y = kx^a$

1.1.2 exponential

1.1.3 circular

4.9 Show the importance of complex numbers in professional engineering calculations Industry sample for mechanical engineering

Conservation of linear momentum during vehicular collisions

Under what conditions can an impact be assumed to be collinear?

Will the impact angle be substantial enough to confirm it is a valid assumption?

Conservation of linear and angular momentum:
http://serc.carleton.edu/sp/library/direct_measurement_video/examples/example9.html

Collinear impacts

- A collinear impact is one in which the longitudinal velocity vector passes through the centre of mass of the impacted object.

Graphical Evaluation of Angle

- The red section on the right, d , is the difference between the lengths of the hypotenuse, H , and the adjacent side, A .
- If the angle θ is very small, then the length of A is very close to the length of H .
- ie, the magnitude of the impact speed approaching along H will be very close to the same magnitude of the collinear impact speed approaching along A .

$$\sin \theta = \frac{O}{H} \approx \frac{O}{A} = \tan \theta = \frac{O}{A} \approx \frac{s}{A} = \frac{A * \theta}{A} = \theta$$

MacLaurin Series for a function f , is:

$$f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \dots + \frac{1}{k!}f^{(k)}(0)x^k + \dots$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots \quad \text{for all } x$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots \quad \text{for all } x$$

$$\tan x = \sum_{n=1}^{\infty} \frac{B_{2n}(-4)^n(1-4^n)}{(2n)!} x^{2n-1} = x + \frac{x^3}{3} + \frac{2x^5}{15} + \dots \quad \text{for } |x| < \frac{\pi}{2}$$

A close up look at the assumption

- where θ is the angle in radians:
- $\sin \theta = O/H$; $H = (A^2 + O^2)^{1/2}$
- $\sin \theta / \theta = \text{Actual} / \text{Longitudinal}$
- At what angle does the error between the longitudinal impact speed and the actual impact speed, based on the angle, exceed 1%? $[\sin \theta / \theta = 99\%]$
- What is the magnitude of the third order term?
- is the third order term significant?

Figure 1. A comparison of the basic odd trigonometric functions to θ . It is seen that as the angle approaches 0 the approximations become better.

$\cos \theta \approx 1 - \theta^2/2$ at about 0.664 radians.
 $\tan \theta \approx \theta$ at about 0.176 radians.
 $\sin \theta \approx \theta$ at 0.244 radians.

- The error between the longitudinal impact speed and the actual impact speed, based on the angle, exceeds 1% at approximately 0.244 radians or 14 degrees.
- An impact angle of 14 degrees, or greater, off normal is usually easy to identify.

Appendix 4: 2014 Formative test trialed at the four Metro Group ITPs

Test Q1: Matrices and simultaneous equations

Students are required to complete below standard questions:

- 1) This question refers to the following system of equations:

$$x - y + 3z = 3$$

$$2x + y + z = 7$$

$$-3x + y + 4z = 9$$

- a) Write the system of equations in the form $A\mathbf{x} = \mathbf{b}$ (1 mark)
- b) Calculate $\det A$, (determinant of A) (2 marks)
- c) Find $\text{inv}(A)$, (inverse of A) (2 marks)
- d) Use any matrix method to solve the system of equations. *Clearly show your process to obtain full marks.* (5 marks)

Test Q2: Industry oriented questions

Students are encouraged to select one of the industry-oriented questions below.

a) A civil industry-oriented application:

In an industrial process water flows through three tanks in succession as illustrated in the figure. The tanks have unit cross-section and have heads (levels) of water x, y and z respectively. The rate of inflow into the first tank is u , the flowrate in the tube connecting tanks 1 and 2 is $5(x - y)$, the flowrate in the tube connecting tanks 2 and 3 is $4(y - z)$ and the rate of outflow from tank 3 is $6z$

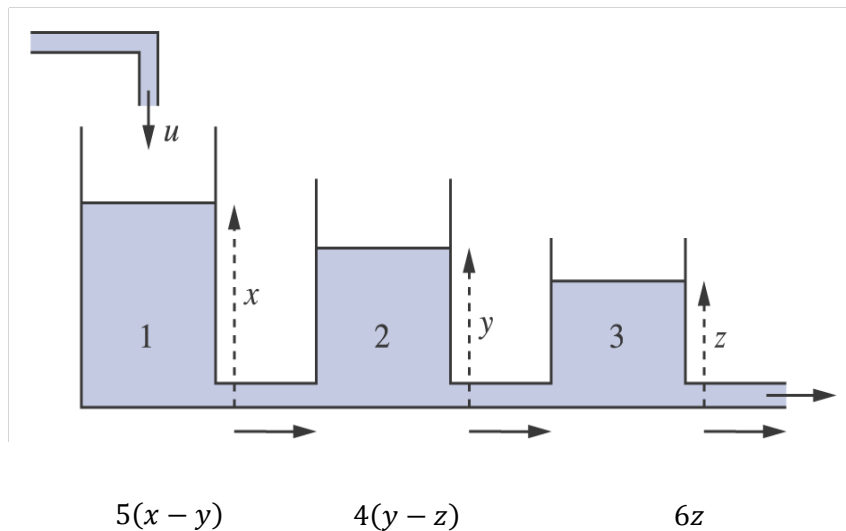


Figure 7 Industrial process: water flows through three tanks

- i. Show that the equations of the system in the steady flow situation are

$$u = 5x - 5y$$

$$0 = 5x - 9y + 4z$$

$$0 = 4y - 10z$$

(2 marks)

- ii. By solving this system of linear equations find x, y and z .

(5 marks)

b) An electrical industry-oriented application:

The figure illustrates an electrical network with mesh currents I_1, I_2 and I_3 shown.

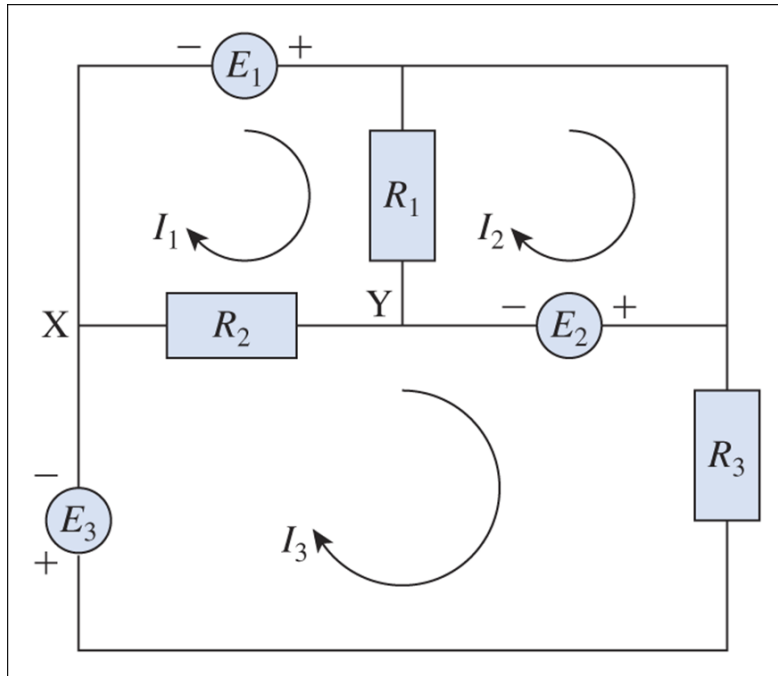


Figure 8 Electrical network with mesh currents

- i. By applying Kirchhoff's voltage law **show** that the matrix equation for I_1, I_2 and I_3 is given by

$$\begin{pmatrix} R_1 + R_2 & -R_1 & -R_2 \\ R_1 & -R_1 & 0 \\ R_2 & 0 & -(R_2 + R_3) \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} E_1 \\ E_2 \\ E_3 - E_2 \end{pmatrix}$$

(2 marks)

- ii. Calculate I_1, I_2 and I_3 given $E_1 = 5 \text{ V}$, $E_2 = 6 \text{ V}$ and $E_3 = 12 \text{ V}$, $R_1 = 15 \Omega$, $R_2 = 5 \Omega$ and $R_3 = 10 \Omega$.

(5 marks)

c) A Mechanical industry-oriented application:

A cantilever beam bends under a uniform load w per unit length and is subject to an axial force P at its free end. For small deflections a numerical approximation to the shape of the beam is given by the set of equations

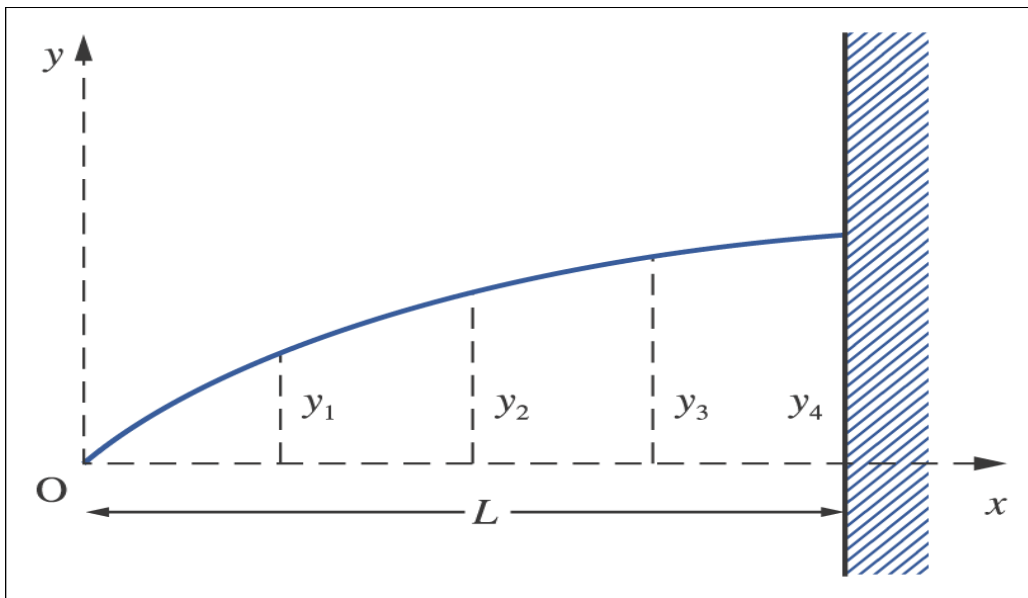
$$-vy_1 + y_2 = -u$$

$$y_1 - vy_2 + y_3 = -4u$$

$$y_2 - vy_3 + y_4 = -9u$$

$$2y_3 - vy_4 = -16u$$

These deflections are indicated in the figure below. The parameters u and v are related to the flexural rigidity, axial load and length of the beam.



cantilever beam bends under a uniform load

Figure 9 A

i. Write the set of equations in matrix form

(1 mark)

ii. Use any method you know to solve these equations when the parameter values are $u = 1$ and $v = 3$.

(6 marks)

Appendix 5: Summary of the answers to question list for focus group with industry leaders

Questions	Civil Engineering	Electrical Engineering	Mechanical Engineering
ALGEBRA			
FUNCTIONS AND COORDINATE SYSTEMS			
Define and graph a relation and its inverse	1,1	1,1	1,1
Convert among common coordinate systems	1,1	1,1	1,1
Graph plane curves in polar co-ordinates	2,1	1,1	3,2
VECTOR ALGEBRA			
Describe a 3 space vector as an ordered triple and in terms of the unit vectors	1,1	1,1	1,2
Perform vector addition, subtraction and multiplication by scalar quantities	3,1	1,1	1,1
Calculate the magnitude and the directed unit vector corresponding to a vector	3,1	1,1	1,1
Find the vector normal and vector tangent to a simple curve at a specified point	1,3	3,1	1,2
Define the scalar product and describe its geometric significance in terms of projections	1,3	3,1	1,2
Apply the scalar product to simple physical problems	1,3	3,1	1,2
Define and find direction cosines	1,3	3,1	1,2
Define and evaluate determinants (up to order 3)	1,3	2,1	3,2
Define the cross product and describe its geometric properties	1,3	2,2	2,2
Verify the distributive law for the cross product	2,3	2,2	2,2
Apply the cross product to simple physical problems	2,3	2,2	2,2
State the equation of a plane	2,3	3,1	2,2
Find a vector N, normal to a plane	2,3	3,1	3,2
Describe the algebraic and geometric properties of the triple scalar product	1,3	3,2	3,2
Apply vectors to engineering applications	1,1	1,1	1,1
LINEAR ALGEBRA			
Apply the rules for matrix addition, subtraction and scalar multiplication	1,3	2,1	3,2
Apply the rule for matrix multiplication	2,3	2,1	3,2
State the additive and multiplicative matrix identities	2,3	2,1	2,2
Investigate the commutative, associative and distributive properties of matrices	2,3	2,1	3,2
Apply the rules governing elementary row operations to obtain an inverse matrix	2,3	3,1	2,2
Show that a consistent set of linear equations may be represented in matrix form and hence find a solution set (using a variety of standard methods)	2,1	2,1	3,2
Understand the term 'row echelon form'	2,3	2,1	2,2
Find an inverse matrix by cofactors	2,3	2,3	3,2
Determine the Eigen values of a matrix	2,3	2,3	3,2
Determine the Eigen vectors for a matrix	2,3	2,3	3,2
Apply Eigen vectors to the linear transformation of Cartesian coordinate systems	2,3	2,3	3,2
Use matrices to solve engineering problems	2,1	2,1	1,2
COMPLEX ALGEBRA			
Convert between Cartesian and polar forms	1,1	1,1	1,1
Represent these forms on a drawing of the complex plane	2,3	1,1	1,2
Apply Euler's formula	1,3	1,1	1,2
Derive the fundamental identity	1,3	3,1	2,2
Verify the complex links between $\sin z$, $\cos z$, $\sinh z$ and $\cosh z$	1,3	3,1	1,2
Deduce and use Osborne's Rule	1,3	3,3	2,2
Define and evaluate the following functions: $\sin z$, $\cos z$, $\ln z$,	1,3	3,3	1,2

sinhz, coshz, zn and combinations of these as appropriate			
Find roots of complex numbers	1,3	3,3	2,2
Show the importance of complex numbers in professional engineering calculations	1,3	1,1	2,1
SERIES			
Understand the term “limit” and the notion of convergence	2,3	1,1	1,2
Use L'Hopital's Rule	3,3	1,3	3,2
Write a Maclaurin Series for transcendental functions	2,3	3,3	3,2
CALCULUS			
DIFFERENTIAL CALCULUS			
Review of differentiation concepts and methods	1,3	2,1	1,1
Perform implicit differentiation	2,3	2,1	1,2
Recognise a composite function and state the Chain Rule for its derivative	2,3	2,1	1,2
Understand the terms concavity, critical point, inflexion, continuity, increasing and decreasing	1,3	2,3	1,2
Define and apply the operator	2,3	3,2	1,2
Apply the Chain Rule for functions of more than one variable	2,3	3,2	1,2
Use partial derivatives to explore 1.4	3,3	2,2	1,2
Apply partial differentiation to solve practical engineering problems	2,3	2,2	1,2
INTEGRAL CALCULUS			
Review of integration concepts and methods	1,3	1,1	1,1
Find an integral by expansion to partial fractions	2,3	3,3	1,2
Find an integral by trigonometric or hyperbolic substitution	2,3	3,3	3,2
Apply integration techniques to find the length of a plane curve	2,3	3,3	1,2
Apply integration techniques to find the area of a surface of revolution	2,3	3,3	1,2
Apply integration techniques to find the volume of a solid of revolution	2,3	3,3	1,2
Apply the method of integration by parts to integrate products of functions (excluding reduction formulae)	2,3	1,3	1,2
Evaluate and use double integrals	2,3	2,3	1,2
Apply integration to solve practical engineering problems	2,3	1,1	1,2
DIFFERENTIAL EQUATIONS			
Define the terms: differential equation, order, degree, initial condition, boundary condition, general solution	1,1	2,1	1,2
Distinguish first and second order Linear D.Es	1,1	2,1	1,2
Recognise that the general solution of a D.E. describes a family of curves and that the particular solution describes a unique curve	1,1	2,1	1,2
Solve a first order linear DE	1,1	2,3	1,2
Solve a homogeneous second order D.E	1,1	2,3	1,2
Apply the above techniques, where appropriate, to find the particular solution for engineering problems	1,1	2,1	1,2
NUMERICAL METHODS			
Construct Newton/Gregory difference tables	1,3	2,3	2,2
Find an interpolating polynomial from data	1,3	3,1	1,2
Use an interpolating polynomial to determine the value of slope at a particular point	1,3	1,1	1,2
Use the trapezium rule	1,1	1,3	1,2
Use Simpson's rule	1,1	3,3	2,2
Apply numerical techniques to solve problems in Calculus	1,1	1,3	1,2
Use an iterative method (such as Euler's method) to solve simple first order differential equations	1,1	3,1	1,2
Know that a numerical process has an associated error bound and that such a bound should be evaluated and along presented with the numerical answer itself	1,3	2,3	1,2

Notes:

- As shown in Appendix 5, the results of the survey are tabulated, where “1” means “Our engineers must know”; “2” means “Our engineers needn’t know” while “3” means “not sure”.
- Note that some results come with “1” and “2” at the same group, which are marked in red in Table 2, indicating the answers from the industry leaders in the same major have an appositive view.
- This insufficient data shows significantly some teaching contents were selected as “Our engineers needn’t know” in a major at the same time. The results come with “1” and “1” mean the same view. Any result with “3” mean one of the views was “not sure”.