Southern Regional Hub-funded project

Project Report

Building Academic Numeracy: Improving Undergraduate Student Outcomes with Proactive Numeracy Learning Support

Report prepared by:
Brigid Casey and Andrea Knowles

Ako Aotearoa

University of Otago
Te Whare Wānanga o Otāgo
New Zealand

Research undertaken at University of Otago

Report prepared by Brigid Casey and Andrea Knowles

Other team members: Chris Linsell, Stewart Hibbert, Clinton Golding, Jon Shemmell

Contact brigid.casey@otago.ac.nz

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Executive summary

This pilot project addressed a growing concern: that incoming undergraduate students have a range of mathematics competency and some experience difficulties with quantitative courses. The project aimed to address low student numeracy through the provision of an innovative numeracy development programme, relevant to the students' discipline context. Early intervention, and active participation in a learning intervention, assisted the transition of first year students into university by improving their conceptual knowledge of mathematics and contributing to their academic success.

The project was designed in three phases. In the first phase, a cohort of undergraduate university students, enrolled in a programme with a compulsory quantitative component, completed a numeracy assessment at the start of the semester. In phase two, students with low numeracy scores were invited to meet with an academic adviser and, if suitable, they were recruited into phase three - an intensive mathematics development programme. A specialist mathematics educator designed the programme to develop students' conceptual mathematics knowledge and constructed an engaging learning environment to build students' confidence, reduce mathematics anxiety, and address affective barriers to learning and using mathematics.

Evaluation of the project included tracking students' engagement and academic performance. Qualitative data from students, academic advisers, lecturers and the specialist mathematics educator informed ongoing improvements to the programme and supported the efficacy of the intervention. The numeracy assessment tool was effective in identifying 'at risk' students and providing individual and cohort information useful for advising and planning the intervention. Students, with a low numeracy score, who attended a consultation with an academic adviser received relevant and personalized advice; assisting them to understand their preparedness in relation to the mathematics requirements of their degree programme. Students who participated in the intensive numeracy development programme developed conceptual mathematics knowledge and skills, contributing to their academic achievement. These students developed a positive attitude towards learning and using mathematics in their studies and outside university.

The project contributes to research on students' mathematical competence and evaluation of the effectiveness of learning interventions. Recommendations for educators and tertiary education organisations include up-scaling and adapting this successful pilot programme to any undergraduate programme.

Implications for practice

This project identified a gap in student learning support and piloted a programme to address this gap. All three phases of the project generated useful information for improving our understanding of student numeracy and providing high quality student learning support. The project identified and addressed the following learner gaps: the 'mathematics problem' (gap between the preparedness of students coming to university and the expectations of lecturers), metacognitive gap (self-knowledge), math anxiety/confidence gap (self-efficacy), and study strategies and skills gap (how to learn).

The potential benefits to learners from the project are described and discussed in detail in later sections of this report. Implications for practice from the perspective of educators and tertiary education organisations (TEOs) is discussed below.

Implications for practice: educators and learning advisers

During the project, the team identified educator gaps and have suggestions for addressing these. The 'mathematics problem', the gap, between the preparedness of students coming to university and the expectations of lecturers, impacts on educators as well as learners.

We recommend explicit communication to intending and current students, including:

- The importance of mathematics;
- The content and level of prior knowledge expected, and the mathematics students can expect in their programme;
- Pre-enrolment advice which include mathematics expectations to ensure students are adequately prepared and enrol in appropriate courses.

Numeracy is implicit in generic graduate profiles that list attributes such as scholarship, communication, critical thinking, information literacy, research, etc. As part of communicating expectations to students, we recommend programme graduate attributes and course learning outcomes explicitly identify numeracy and/or mathematics competencies, where applicable.

Many students assume that if they have the numeracy credits for university entrance they are adequately prepared for university level quantitative work – this is not always the case. Therefore, the message needs to be clear to high school students from Year 9 that mathematics is important for many tertiary programmes. This unoriginal recommendation requires support from educators, parents and schools' advisers.

Educators and advisers have access to data useful for understanding student learning needs. However, data needs to be used wisely. Students starting a degree may have many things in common, but they are individuals with distinct and diverse needs and circumstances. Making generalisations from the cohort data ignores these individual differences. The numeracy assessment tool data was not only useful for identify low numeracy students; detailed individual reports, used to personalise advice and focus learning support efforts, were valuable for recruiting and engaging students in the intervention.

The project initiated good communication between lecturers and the specialist mathematics educator (SME). Lecturers provided advice on the mathematics content of their academic programmes and their expectations of students; the SME provided useful feedback to lecturers, to inform curriculum planning and design. The SME and academic adviser, with the cooperation of academic staff, were able to access information and resources about the programme to contextualize the mathematics development programme and customise advice to students.

Throughout the project, and the dissemination of the results via presentations and direct communication with academic staff, we identified and addressed some common misconceptions about low numeracy students. Although staff sensed a problem, they were previously unaware of high proportion of low numeracy students and that these students were not necessarily low ability students. An attitude, expressed by some staff, was that if students did not have the requisite numeracy competency by the time they started tertiary education it was too late to learn. Clearly, this is not correct. Other comments from staff that display misconceptions about mathematics learning included, "students just need to fail" and "all students need is some tips and tricks to pass". More work is required to understand and address staff misconceptions and attitudes. In the meantime, we recommend that everyone communicating with students adopt the message that "mathematics is important and can be learned".

Implications for practice: tertiary education organisations

We identified TEO gaps and opportunities during this project. Although it was beyond the scope of this research to identify the extent of mathematics learning support available across the tertiary education sector in New Zealand, from anecdotal evidence and the international literature, it is reasonable to assume limited support is available. What support is available is likely to address procedural mathematics knowledge only, which may improve students' academic success in the short term while pushing the problem of low numeracy students to the next academic level. This is an area where further research is recommended, and good practice identified and disseminated across the sector.

The economic argument for providing numeracy support is the retention and progression of students in programmes with compulsory quantitative components. The fee income from a small number of students retained by the TEO, who would otherwise be lost due to withdrawal or failure, would offset the cost of the intervention. Further benefits to the TEO accrue from improving educators' understanding of the mathematics abilities of their students, as well as improving the teaching and learning experiences of educators and mathematically well-prepared students. The expert advice of a specialist mathematics educator can assist educators to develop strategies to embed numeracy in the curriculum and further support student learning.

Introduction

This pilot project "Building Academic Numeracy" addresses a growing concern for the "mathematics problem": that there is a mismatch between the numerical competency of students upon entry, and the expectations of university teachers (Marr & Grove, 2010). Poor numeracy is an issue for undergraduate students across a range of disciplines (Galligan & Hobohm, 2015; Hodgen, McAlinden & Tomei, 2014; Matthews, Croft, Lawson & Waller, 2013; Linsell & Anakin, 2012). A study investigating business students' numeracy in a compulsory 100-level statistics course at a research-intensive university (Casey, 2015; Linsell & Casey, 2013) found 25% of the students had numeracy levels below that expected of a competent Year 9 student (13-year-old). While there was a highly significant relationship between low numeracy and failing quantitative papers, there was no relationship between low numeracy and failing non-quantitative papers. Students with low numeracy are not necessarily low ability students; they lack specific conceptual knowledge and skills needed for quantitative work at university level. High fail rates in first year quantitative papers also indicated many students were inadequately prepared for their courses.

The issue with mathematics transition from secondary school to university is complex. Hong et al. (2009) recognized that a lack of understanding about different teaching and learning styles contribute to the problem. It seems that schools value procedures; whereas, universities emphasise "concepts, mathematical thinking, applications and problem solving." (Hong et al., 2009, p.888) However, teachers and lecturers may be unaware of the differences. University lecturers and tutors are unlikely to have a background in mathematics education and may not have the pedagogical skills to address the under-preparedness of incoming students and develop students' conceptual understanding of mathematics (Walsh, 2017). A specialist mathematics educator who understands the students' past experiences in a school setting and the potential limitations of their mathematical knowledge, along with their university programme requirements, is well placed to assist students to make a successful transition from school to university.

Students who underperform in quantitative papers may be experiencing difficulty transferring mathematical knowledge and skills across disciplines (Brogt, Soutter, Masters, & Lawson, 2014). "Building Academic Numeracy" complemented the transfer approach by identifying and addressing gaps in students' understanding, thereby supporting the transfer of knowledge to new contexts.

There is also concern that students' mathematics competency is declining and there is a growing portion of students 'at risk' of failing courses (Rizwan, & Alsop, 2016; Faulkner, Hannigan, & Gill, 2010). Indeed, mathematics is considered the strongest predictor of progression in higher education institutions (HEA, 2010, as cited in Walsh, 2017).

The "Building Academic Numeracy" pilot project aimed to address low levels of numeracy in undergraduate students and improve student achievement, through the provision of a pilot intervention that identified 'at risk' students and developed their conceptual knowledge of mathematics.

The following definitions clarify the distinction between conceptual and procedural knowledge. Procedural knowledge is when students have knowledge, but not necessarily an understanding, of mathematical rules, algorithms, procedures and processes. It is like a tool box that includes facts, skills and methods (Barr et al., 2003). According to Rittle-Johnson and Alibali (1999, p.176), "Simply using a correct procedure often does not lead to a better understanding of the underlying concepts." On the other hand, students' conceptual knowledge is rich in understanding, relationships and connections. It cannot be learned by rote as procedural knowledge can often be; but involves thinking and metacognitive reflection. It is described as the "ideas, relationships, connections, or having a 'sense' of something." (Barr et. al., 2003).

The Tertiary Education Commission (TEC) defines numeracy as "the bridge between mathematics and real life. A person's numeracy refers to their knowledge and understanding of mathematical concepts and their ability to use their mathematical knowledge to meet the varied demands of their personal, study and work lives." (2009, p. 59). The Ministry of Education adopted the definition of numeracy as having "the ability and inclination to use mathematics effectively in our lives – at home, at work, and in the community." (Fancy, 2001, as cited in Neill, 2001, p.3) For this project we adopted Dr. Chris Linsell's definition: "In an undergraduate context, to be numerate is to have the knowledge, skills and confidence to use mathematical tools in a range of disciplinary contexts." However, as we worked with students it became clear that motivation and the inclination or willingness to learn and use mathematics, was also critical to success.

Maths anxiety features strongly in the literature on mathematics education and numeracy learning support. A student's attitudes, beliefs and emotions impact on how well they learn and use mathematics (OECD, 2013, as cited in Attard, Ingram, Forgasz, Leder, & Grootenboer, 2016). Maths anxiety is linked with perceived self-efficacy (Vallavicencio, & Bernardo, 2013) and lack of confidence, which can impact negatively on learning mathematics (Hodgen, McAlinden, & Tomei, 2014).

In designing "Building Academic Numeracy" we drew on Dweck's (2012) research that promotes the growth-mindset concept - that mathematics competency is not innate, but it can be developed, with effort. As well as developing conceptual knowledge of mathematics and procedural skills necessary for academic success, we aimed to enhance student confidence and address affective barriers to learning and using mathematics. To further assist successful transition to university, we aimed to develop students' metacognitive and study skills.

The project was designed to be flexible and responsive to individual learners' needs, including diverse cultural identities (Alkema & Rean, 2013; Airini, et al., 2009; Greenwood & Te Aika, 2008). This was achieved by fostering collaboration; encouraging learners to support each other; and acknowledging students' contributions. A warm welcome, shared kai (food) and social interaction encouraged a sense of belonging and contributed to student engagement in the programme.

Student numeracy is an issue across the tertiary education sector both in New Zealand and overseas. From the literature, learning support for students studying mathematics and statistics is variable and generally limited to drop-in centers and appointment-based models (Marr & Grove, 2010; MacGillivray, 2009; Matthews, Croft, Lawson & Waller, 2013). In these models, student peer tutors, academic staff, or student learning advisers provide assistance with procedural solutions to one-off problems or assessment questions. This is different from the current project where students with low numeracy were proactively identified and offered an opportunity to develop a strong foundation in mathematics required for university level work and beyond. In this model, a specialist mathematics educator developed students' conceptual understanding of mathematics, required for quantitative reasoning, along with factual knowledge and procedural skills. For these reasons, other models of student support may complement our programme, but they were not a substitute.

At the university where the pilot was introduced, extra support for undergraduate students to develop their mathematics competency was limited to extra tutorial or help sessions and Peer Assisted Study Sessions (PASS) in a small number of 100-level papers; some online 'self-help' resources but no specialist learning adviser available for consultations or drop-in help sessions.

See table one for a comparison of the potential student learning effects of the intensive, co-curricular numeracy development programme piloted in this "Building Academic Numeracy" project and other types of mathematics learning support.

Table 1: Summary of mathematics support and student learning development

Mathematics learning support	**Develops students'**				
	Conceptual knowledge	**Procedural knowledge**	**Confidence**	**Meta-cognition**	**Study skills**
Intensive, co-curricular programme (e.g. Building Academic Numeracy)	**Yes**	**Yes**	**Yes**	**Yes**	**Yes**
Intensive, pre-enrolment programme	No	Yes	Maybe	Maybe	Yes
Peer Assisted Study Sessions (PASS)	No	Yes	Yes	Maybe	Yes
Extra tutorial/ help sessions	No	Yes	No	No	No
1-1 consultation with maths learning adviser	No	Yes	Maybe	Maybe	Yes
Drop in maths support	No	Yes	No	No	Maybe
Self-help resources (e.g. online modules)	No	Yes	No	No	No

In summary, the pilot intervention “Building Academic Numeracy” offered:

- Provision of a numeracy development intervention for undergraduate students in a programme with compulsory quantitative courses
- Diagnostic assessment which raises students’ awareness of the gap between their competency, the requirements of their programme, and the expectations of university lecturers
- Proactive, early alert and intervention supported by data and analytics
- Student engagement with academic advising and learning support before they underperform
- Specialist mathematics educator who creates a responsive and supportive learning environment
- Development of conceptual mathematics understanding which enhances students’ ability to transfer knowledge and skills across disciplines and unfamiliar contexts
- Improved student confidence and metacognitive skills that boosts persistence with quantitative courses and help-seeking behaviour
- A means to address ‘maths anxiety’ and potentially deliver life-long benefits

The project “Building Academic Numeracy”

Three phases: description, results, discussion and recommendations

The “Building Academic Numeracy” pilot project comprised the following three phases: phase one - a numeracy assessment; phase two - personalised academic advice; and phase three - an intensive, co-curricular numeracy development programme which we called: “Mathematics for University” (M4Uni). (See figure one)

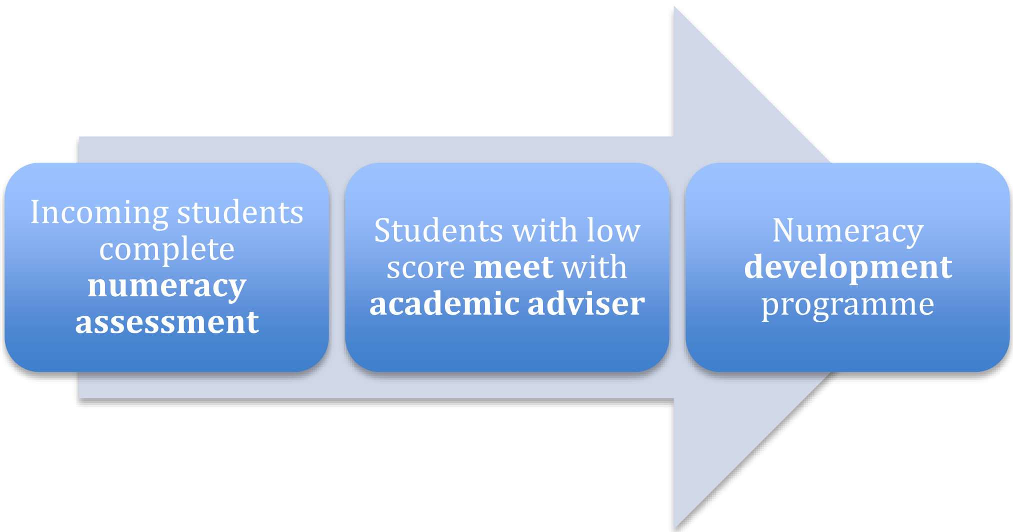


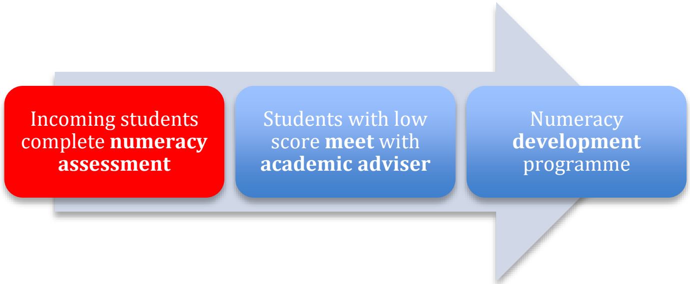
Figure 1: "Building Academic Numeracy" three-phase design

The pilot project targeted first year students enrolled in the Bachelor of Physical Education (BPhEd) programme. Students admitted to the programme in 2017 had met the numeracy requirement for university entrance (UE). The BPhEd does not have an entry numeracy requirement above the current New Zealand Qualifications Authority UE minimum of 10 credits at NCEA level one or above, made up of achievement standards available through a range of subjects, or a package of three numeracy unit standards.

There were 246 students in the target BPhEd student cohort: 48% female, 53% male, 18% Māori, 5% Pasifika and 4% English as an additional language. A feature of this particular cohort is the relatively low number of international students. We discovered that many of the BPhEd students were high achieving sports people with well-developed motivation, persistence and goal-orientation attributes. The staff and academic leaders in the department enthusiastically supported the project; reflecting a student-oriented culture that values teaching and learning.

The next sections of this report describe each phase of the project starting with a description including objectives, context and background where relevant, and the approach or methods adopted. The description is followed by results, discussion and recommendations.

1.0 Phase one: Numeracy Assessment



1.1 Description of phase one

The purpose of the assessment was to identify students with low numeracy and generate information to inform and strengthen the learning intervention. The Tertiary Education Commission (TEC) Literacy and Numeracy for Adults Assessment Tool (TEC, 2013) is an online, adaptive tool that provides "robust and reliable information on the reading, writing and numeracy skills of adults" (TEC, 2016, para. 2). We adopted the general numeracy assessment section of the TEC tool, which includes questions on number knowledge, number strategies and measurement. The assessment comprises 30 questions drawn from hundreds of assessment items, using New Zealand contexts. 'Adaptive' means the computer alters the difficulty of questions, in response to the learner's answer – correct answers lead to harder questions until the learner's level of competency (i.e. score) is reached. Because the tool is adaptive, it is not constrained by the ceiling effect; it is unusual for students to achieve a perfect score.

There is a range of reports available in the tool. Immediate, individual results allow learners to identify their strengths and areas for development. These detailed reports were helpful for the academic adviser and the specialist mathematics educator to gauge what individual students knew or did not know, to build on a students' strengths and not focus solely on gaps in their knowledge. Cohort reports are also available, providing information useful for student advising, early intervention programmes, and curriculum development.

At the beginning of the academic year, students enrolled in a compulsory 100-level BPhEd paper were asked to complete the TEC numeracy assessment. In a preliminary lecture, students were informed about the "Building Academic Numeracy" pilot project and the numeracy assessment tool. They were given written information and invited to sign an ethics consent form. The presentation also highlighted the importance of mathematics in their programme and for future employment. (See figure two: example slide from the presentation.) To alleviate test anxiety, supportive language was used and students were encouraged to give the assessment their best shot.

This process was repeated again at the beginning of the second semester, providing another opportunity for students to join the project.

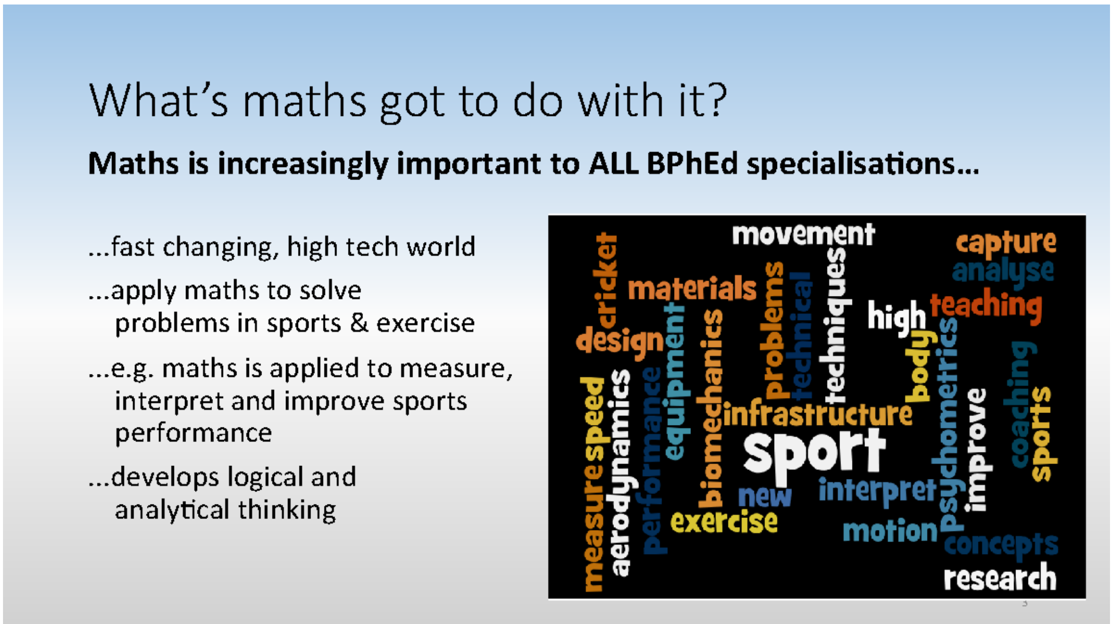


Figure 2: Information slide from the preliminary lecture presentation

1.2 Results of phase one

Combining data from semester one and two, 221 students completed the numeracy assessment tool. Thirty percent of the cohort had a numeracy score (below step 6 or approximately 690/1000) indicating a numeracy competency approximately equivalent to, or less than, level 4 of the New Zealand curriculum: the goal for Year 8 or the final year of schooling in the primary sector. See table two for a summary of the numeracy assessment results.

A threshold score for intervention was set at 740/1000. Previous investigations of undergraduate student numeracy using the TEC assessment tool, and comparing student grades in quantitative courses, informed this decision (e.g. Linsell, Casey, & Han-Smith, 2017). In total, 118 students scored below this threshold, of which 27 were Maori and seven Pasifika. All these students were individually followed up and invited to a one-to-one consultation with an academic adviser.

	Students Complete diagnostic	Average score out of 1000	Students score below step 6	Students Score below 740 threshold	Percentage of students below threshold
All students	**221**	**746**	**52**	**118**	**53%**
Female	108	741	28	59	55%
Male	113	751	24	59	52%
Māori	37	695	18	27	73%
All Pacific Peoples	10	738	2	7	
English not 1st Language	10	729	3	8	

Table 2: Numeracy assessment results

At the end of the pilot project, students' numeracy assessment scores were compared to their final grades in two 100-level, semester two, BPhEd papers with quantitative content. These were not specifically mathematics papers but required students to apply mathematical concepts such as interpreting graphical and numerical data to solve problems.

Students who scored above our threshold of 740/1000 and received no further intervention generally performed well in the quantitative papers (112 students completed a total of 128 papers). In comparison, students who scored below the threshold and either did not participate in the numeracy development programme or attended fewer than four sessions (low attenders) generally performed less than satisfactorily (100 students completed 115 papers). Chart one displays this comparison.

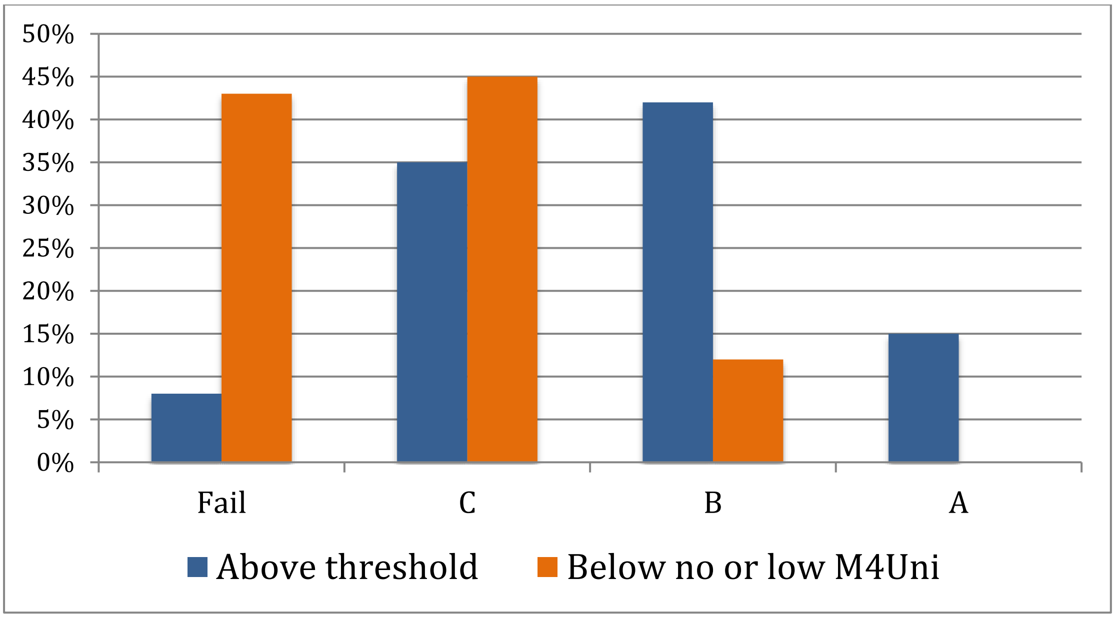


Chart 1: Comparing student numeracy with grades in quantitative papers

Over 90% of the cohort (221 of 246) completed the TEC assessment tool but there were 25 'hard to reach' or 'missing' students who did not. Only ten of these students completed either or both the semester two quantitative papers (paper count = 18) and the grade profile was similar to that of the students who scored below the threshold. (See chart two)

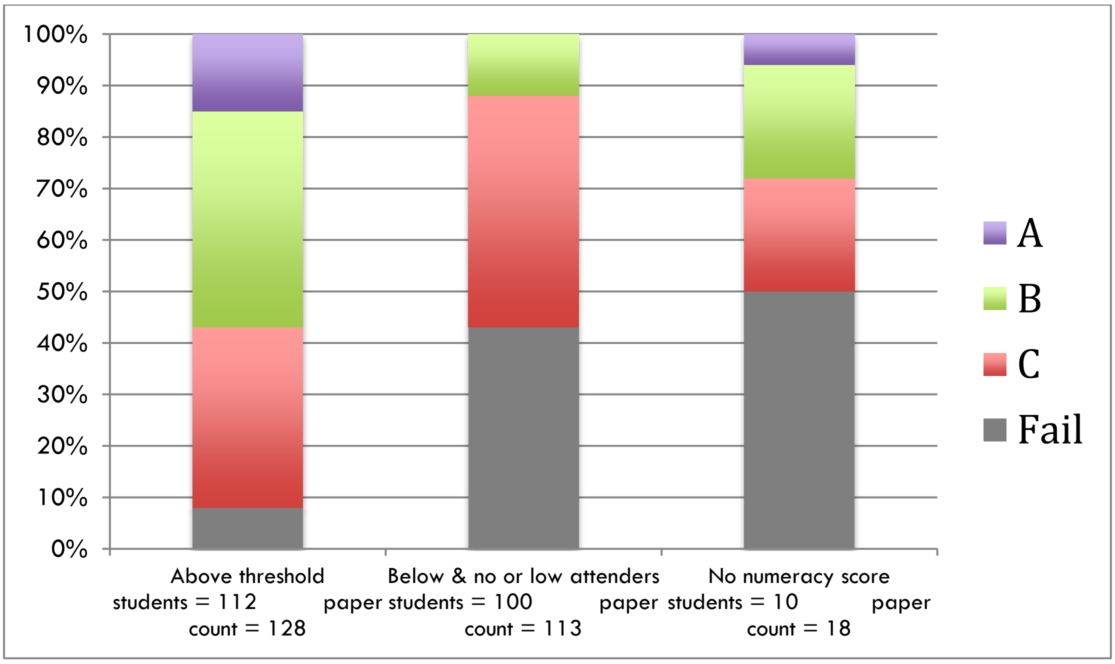


Chart 2: Comparing grade profiles for high, low and no student numeracy scores

Priority learners

A feature of this cohort was a relatively high number of Māori, and low number of Pasifika, students. Māori students comprised 15% of the total cohort. Eight of the 25 students who did not complete the assessment identified as Māori. The proportion of Māori students whose numeracy score was below the threshold for intervention (73%) was higher than the total cohort (53%). However, of the 27 Māori students below the threshold, 12 engaged with the next phase of the intervention (consultation with an academic adviser). This was 44% of the eligible Māori students, compared with 37% for all students who met the criteria. This higher rate of engagement continued through to phase three: the numeracy development programme.

1.3 Discussion of phase one

Early assessment of students' numeracy provided timely information for students, educators and academic advisers. Running the numeracy assessment early in the semester addressed problems faced by student learning interventions relying on poor performance in course assessments to identify 'at risk' students. At this later stage, students' self-efficacy and motivation may be fading and there is less time available to run an effective learning development programme.

The TEC numeracy assessment tool was fit for the purpose of identifying 'at risk' students. Students who achieved a high numeracy score were most likely to pass quantitative papers with a high grade; students with a low numeracy score were much more likely to fail these papers. Detailed reports generated by the assessment tool provided diagnostic information to support and strengthen the intervention.

The TEC tool is widely adopted in the tertiary education sector in New Zealand. It has very good reliability and validity (TEC, 2016). We adopted it for these reasons plus it identified gaps in students' conceptual knowledge rather than assess specific procedural mathematics knowledge required for 100-level courses. In other words, developing a pre-entry assessment to identify students' procedural skills aligned to the level and content of first-year courses would not be 'fit for purpose' for this intervention and it would require resources better expended on student learning development.

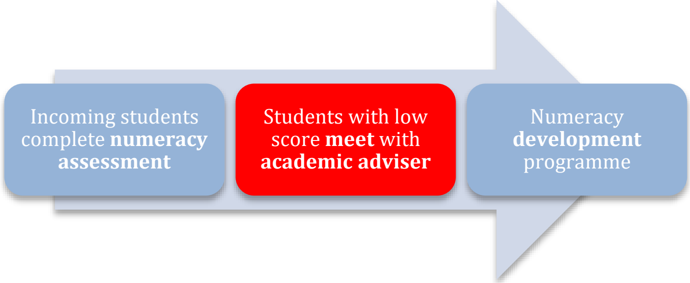
1.4 Recommendations from phase one

The TEC general numeracy assessment tool is recommended as it is valid and reliable, and fit for the purpose of identifying students' numeracy competency for early intervention and numeracy learning support. Reports are immediate, detailed and useful for student advising and informing the design and development of learning intervention programmes.

Timing the assessment close to the start of the semester or academic year is recommended.

It is also highly recommended that completing the assessment tool is not optional for students. It is likely the students most reluctant to complete a numeracy assessment are the ones who would benefit most from the follow-up and intervention.

2.0 Phase two: Personalised Academic Advice



2.1 Description of phase two

Students with low numeracy scores were invited to meet with an academic adviser to discuss their individual numeracy report and, if appropriate, recruit them into the numeracy development programme. These approximately fifteen-minute, one-to-one meetings were held in a central location, away from the department.

The purpose of the meetings was to assist students to interpret their report and discuss their mathematics preparedness for their studies. The adviser used language consistent with the growth-mindset model and reinforced the idea that mathematics, rather than being a fixed trait, can be developed with effort. The adviser discussed details in the report that identified the student's strengths and where further development was recommended.

The adviser asked open-ended questions to explore the student's experience and attitude towards mathematics, their academic goals, and their availability and time management skills. This conversation helped inform the adviser's recommendations for learning support. Students with low numeracy, clear academic goals and good time management were actively recruited into the next phase of the project - the numeracy development programme, Mathematics for University (M4Uni). Students keen to engage with the programme were given information so they had realistic expectations and understood the commitment involved.

All students, including those unable or unwilling to engage further with the project, were encouraged to attend lectures and tutorials, keep up with their courses, and make use of the limited support available. Students were encouraged to seek help when they needed it and to consider support as a normal part of the education process.

The concept of proactive or 'intrusive' advising best describes this phase of the project. This is an action-oriented model of advising that involves and motivates students to seek help when it is needed (Rodgers, Blunt, & Trible, 2014). By providing relevant and personalised information, the adviser helped students to accurately judge their mathematics ability in relation to the demands of their degree programme. The discussion assisted students to develop metacognitive skills (to better understand, monitor and evaluate their own learning) and, in some cases, create awareness of the need for learning support to achieve their academic goals. During this consultation phase, some students reported that they would not have otherwise sought learning support without their numeracy assessment results from phase one of the project – they were unaware of the gap between their preparedness and the expectations of university teachers.

Direct communication and a personalised approach facilitated a good rapport between the student and the academic adviser. With information and advice, students were able to prepare a learning support plan to suit their requirements and circumstances; they were encouraged to be proactive in addressing their own learning development needs. Even students who declined to join M4Uni benefited from holistic academic advice and an early connection to student learning support.

2.2 Results of phase two

Of the 118 students with numeracy scores below the threshold for intervention, 44 attended a consultation with an academic adviser. A small number of students presented with low numeracy scores but were not necessarily lacking the conceptual mathematics understanding that M4Uni aimed to address. Some students reported rushing to finish the assessment by randomly selecting answers or being put off by unfamiliar context or misreading the questions. However, the programme could help students achieve academic success in other ways. For example:

- An international student, with English as an alternative language, was numerically competent but was interested in joining M4Uni to improve their English vocabulary associated with mathematics.
- A mature student had not studied mathematics for some years and wished to join the programme to revise, become more confident and improve their quantitative literacy. They were keen on being part of a cohesive, helpful and supportive group.

There were a number of students who attended a consultation but were either unavailable or chose not to join M4Uni. Some of the reasons for students not engaging are illustrated in the following examples:

- Student has had a block with mathematics since primary school and could see M4Uni being a real help but could not come to M4Uni due to sporting commitments.
- Student does not like mathematics: not so keen to join intervention.
- Student has not studied mathematics for two years but unable to attend M4Uni due to employment outside university.
- Student already failed quantitative papers: choosing different programme because not confident in mathematics.

Talking to students about their mathematics backgrounds and attitudes was illuminating and provided useful information for the academic adviser and the specialist mathematics educator. Two broad categories emerged: students who recognised they struggle with mathematics and acknowledged maths anxiety, and those who studied mathematics up to level two or level three NCEA and were unaware they had a ‘mathematics problem’. Another theme that emerged in the consultations was that some students were unaware of the occurrence and level of mathematics in their courses.

2.3 Discussion of phase two

The one-to-one consultation with an academic adviser was a key step in the intervention. The numeracy assessment was useful for identifying ‘at risk’ students and the individual report provided information to start a conversation between the student and the adviser.

The aims of the consultation included assisting students to interpret their numeracy report and develop a realistic learning development plan. The adviser aimed to find out more about the student's mathematics background and learning needs; and assess their suitability for M4Uni. It was also an opportunity to inform students about M4Uni and recruit them into the programme.

2.4 Recommendations from phase two

The academic advising phase of the intervention is essential for success. However, it is resource intensive with implications for up-scaling the project, especially in tertiary education organisations where academic advising is not recognized and it is not normal practice. An 'academic coaching' model (McClellan & Moser, 2011) is recommended where advisers have an understanding of the curriculum associated with the student's programme of study and focus on strategies for success. Advisers encourage students to take responsibility for learning and assist them to problem solve. One-to-one consultations assist advisers to understand the challenges for students.

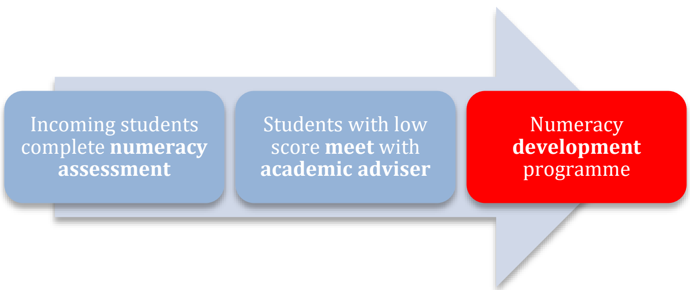
The timing of the advising appointment was critical. Early contact with students and timely, personalised academic advice enabled students to engage in learning support and adopt strategies to be academically successful before they suffered academic setbacks.

Because there is no ceiling effect, the assessment tool also identifies very able students who may benefit from academic advice or academic mentoring. For example, a student who loves mathematics, but has been advised by their parents to study a 'professional' degree, may be unaware they can study mathematics alongside their degree major. They may be unaware of disciplines, such as finance, that are not taught in schools but have well-defined career pathways.

At the start of their first year at university, some students may not have the maturity or motivation to opt into a voluntary, co-curricular learning intervention such as M4Uni. However, they may be ready to engage with support at a later date. Ideally, numeracy development support should not be limited to first year students but available for all students.

3.0 Phase three: Numeracy Development Programme

Mathematics for University (M4Uni)



3.1 Description of phase three

The initial stages of the project involved investigating students' numeracy, identifying students with low numeracy, and understanding their challenges and learning development needs. The final stage involved designing and delivering an intensive numeracy development programme: Mathematics for University (M4Uni). M4Uni aimed to develop students' confidence and provide them with a strong base of conceptual mathematics knowledge and skills to build on throughout their degree programme, and beyond.

M4Uni adopted a pedagogical strategy, informed by research and practice, involving active learning; strong learner/educator partnerships; and small group, peer and individual learning. It was modelled on a successful programme offered at the university's College of Education.

There is no quick fix; developing conceptual understanding takes time (Alkema & Rean, 2013). Therefore, M4Uni started in week three of the semester and was delivered in weekly, two-hour sessions over the full academic year.

The specialist mathematics educator (SME) role was fundamental to the success of the programme. The SME, a qualified mathematics teacher with experience in the secondary and tertiary education sectors, had expert mathematics pedagogical content knowledge and understood how students develop mathematics ideas. The SME emphasised mastery of concepts, over memorization and skills, to lay a foundation for further learning. (See appendix one for a summary of the M4Uni content)

M4Uni was learner-focused – students came before content. Although planned, each session was flexible and adapted to the students, and their needs, on the day. Each student had different and evolving cognitive and emotional maturity, levels of motivation, resilience, and perseverance. Prior knowledge was respected, and students were encouraged to build on what they already knew. The key to keeping students engaged and learning was building on their strengths, rather than solely focusing on gaps in their knowledge. Throughout the project, a growth mindset-model was purposefully adopted, and supportive language used, to build confidence, perseverance and a can-do attitude to learning mathematics.

M4Uni was contextualized to the BPhEd programme by linking mathematics concepts to the physical education discipline through applications and examples. The SME consulted lecturers and teaching resources to become familiar with the mathematics requirements of the programme and students were encouraged to look for connections between the curricula of different papers they were studying. Students were also encouraged to think outside their programme of study, to make connections between their coursework and other aspects of their lives such as paid employment or sports.

Throughout the year, the SME attempted to provide students with an engaging and enjoyable learning experience. Students collaborated with the SME and with their peers, in a friendly, energetic and inclusive environment. In focus group sessions, students reported that the small group size, humour, and interactive learning contributed to the success of M4Uni. The learning environment was deliberately constructed to put students at ease and create a sense of belonging. In personal communications, students expressed their appreciation of generous hospitality, a warm welcome, and food at every session. These small gestures, and a central location away from the department, signalled a distinctive learning experience and created an environment where students were comfortable to display their knowledge, or lack of, and ask questions. Students felt their contribution to the learning environment was important and valued.

Monitoring and evaluation

M4Uni was monitored and evaluated to inform improvements and to provide evidence of its efficacy. Academic performance was evaluated by comparing the grades, in two quantitative papers, of students who participated in M4Uni with students who had similar numeracy assessment scores but either did not participate or attended fewer than four M4Uni sessions. Improvement in mathematics competency was also monitored during the programme through regular diagnostic and summative quizzes (pre-test and post-test) and ongoing formative assessment.

Regular reflections from the specialist mathematics educator, an online student survey (n=14), student focus groups, observation, and informal feedback, enhanced our understanding of the outcomes for students and improved our confidence in the quantitative measures of academic performance.

3.2 Results of phase three

Student engagement and participation

In total, 18 BPhEd students were recruited into M4Uni. Although the recruitment process took into account students' willingness and availability to take on this extra commitment, some students who started M4Uni withdrew after a few sessions because they struggled with their academic workload and other demands on their time. One student attended regularly but left university halfway through the year to take up an employment opportunity.

Improved academic performance

M4Uni was successful in providing students with the opportunity to develop the knowledge, skills and attitudes needed to succeed in quantitative papers.

Nine BPhEd students who attended between six and fifteen sessions comprised the group for evaluating M4Uni's impact on academic performance. Eight students completed papers (n=14) with quantitative content in semester two. Although the number was small, the grade profile of participating M4Uni students was similar to the group who scored above the threshold for intervention (See chart three).

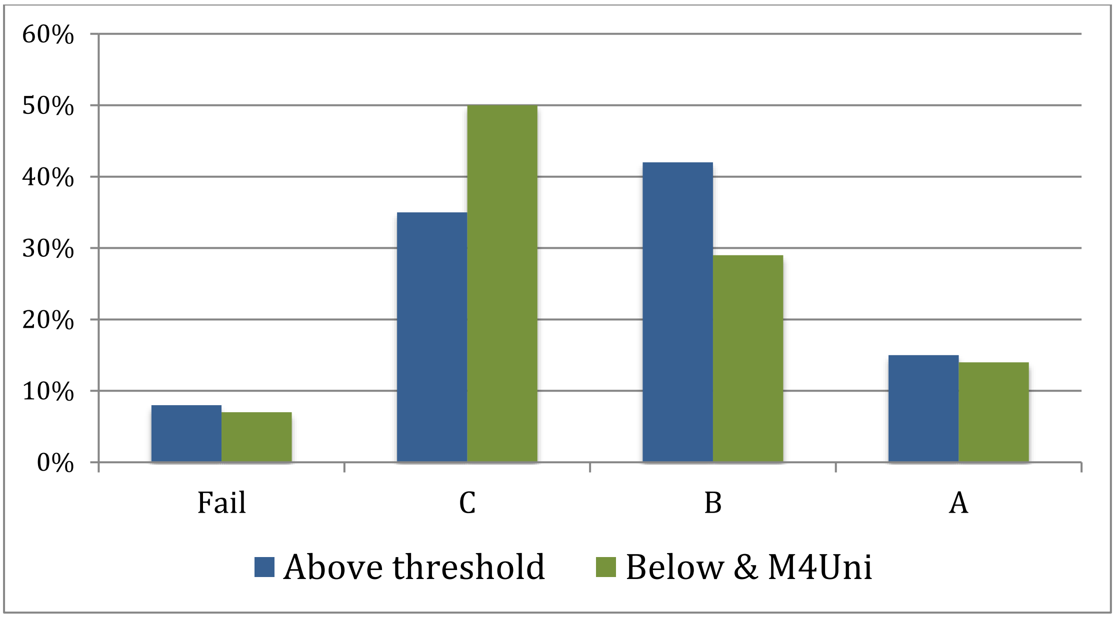


Chart 3: Grade comparison of students with numeracy scores above the threshold, and M4Uni students

This indicated a significant improvement compared with low numeracy students who either did not participate in M4Uni or attended fewer than four sessions. (See chart four)

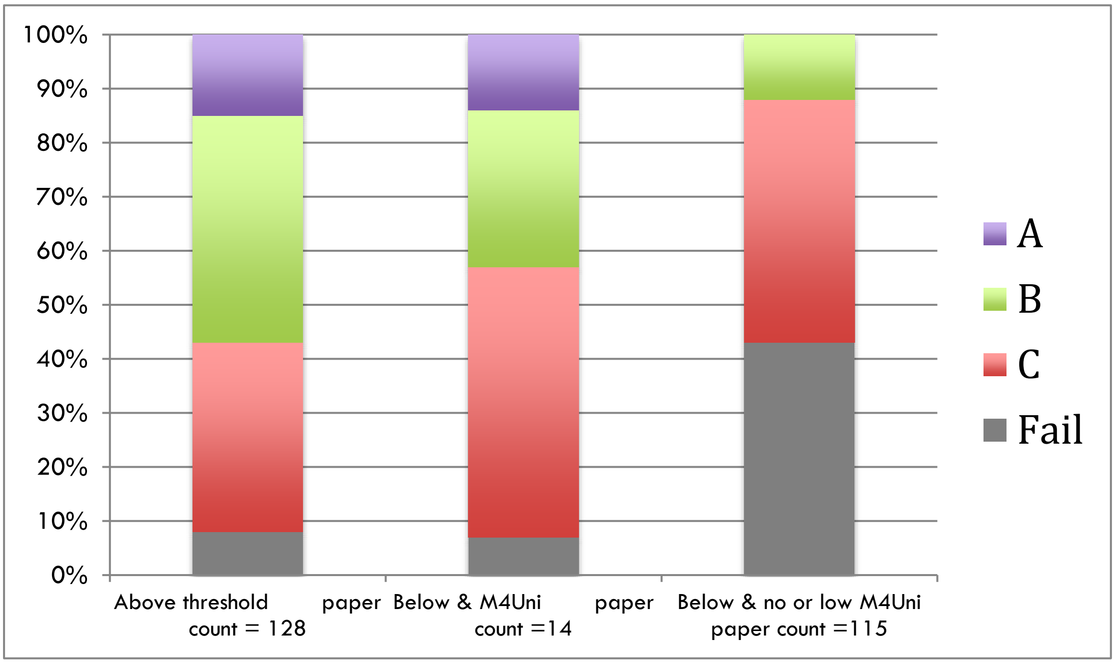


Chart 4: Grade profile comparisons

Improved mathematics knowledge, mathematics vocabulary, understanding concepts, and solving problems

The SME's detailed weekly reflections and written observations documented students' developing mathematics knowledge; building from basic concepts to applications in discipline and unfamiliar contexts. For students who started M4Uni with little number knowledge or understanding of measurement and place value, this was a long process.

Some students did not initially recognise problems as being related to mathematics and we observed students starting out in M4Uni had difficulty translating mathematical equations into words or reading a question and picking out the relevant information to put into a formula. This changed over time as students started to get a sense of number – they started questioning their own answers and asking themselves: "Is this answer remotely correct?"

Initially, some students had clearly observable barriers to learning mathematics. For example, their eyes would glaze over and they would visibly switch off when they saw too many numbers. This changed as students gained basic knowledge and worked with numbers. For example, during session 13 on algebraic thinking, the SME observed that a student working on solving algebraic equations involving fractions made good progress when the concepts were stripped back. As the session progressed, this student appeared to enjoy being able to work on harder problems that required a basic understanding of fractions, decimals and algebraic thinking.

Students gradually wanted to understand the 'how' and explore 'why' - not just accept the right answer to a problem. For example, the SME wrote after session 15, "I have seen a change in [the students'] ability to question and to want to explore why we do things as opposed to accepting maths as some kind of magic that is too difficult to understand." Eureka moments became more common, as in the follow quote from a student:

"I can actually understand it!"

Helped with academic work and life outside university

Students were encouraged to connect their learning to their academic discipline, building on the basic concepts to more complex learning required in their courses. Students were also encouraged to use mathematics effectively in their lives outside university. 67% of the survey respondents agreed M4Uni helped significantly with university work and with life outside the university. Students expressed an increased awareness of financial literacy; they felt better able to make informed choices regarding money.

Activities used in the M4Uni sessions were not always directly related to course content. However, these activities were well thought out so that they provided a stepping-stone to allow students to construct their own understanding in future learning. In the survey, 83% of the respondents said M4Uni would make a difference to their future study.

M4Uni students said…

"I used to look at lecture slides and go to tutorials and not even know what they were talking about…now I can follow."

"I go to the supermarket and compare prices, looking for better value. My flatmates think I'm weird but I'm spending about $20 a week less on food than they are."

Increased confidence and engagement in learning, including metacognitive skills

Of the medium to high attenders of M4Uni, all the students agreed that there was an improvement in their mathematics confidence, especially using mathematics in situations such as employment or shopping. M4Uni helped students feel confident to take risks and become more open to learning new methods rather than saying "I don't understand".

As the year progressed, students exhibited greater confidence in their ability to reason and question. We observed students taking risks with their learning, questioning misconceptions and readily seeking help. Students began monitoring and evaluating their own learning; they became more aware of, and wanted to fill, the gaps in their knowledge. Initially they were unaware of the hierarchical nature of mathematics and the scaffolding necessary for learning, but this changed over time.

M4Uni students said…

"I am much more confident and I try to challenge myself to do it without a calculator."

"I feel so much more confident with algebra now. I always found it impossible with all the letters but it's not that bad really."

"I never understood that in school, now I realize I was making it too complicated – I just needed to understand it."

Perseverance and resilience

Although the number of students who participated in M4Uni was small, these students demonstrated perseverance, kept up with their studies and completed the quantitative courses in their degree programme. They brought course material to the M4Uni sessions and discussed it with the SME, so they would know what questions to ask the lecturer. Students mentioned they felt more able to approach lecturers for help on specific aspects they were struggling with. One student said that the ongoing support from the SME and academic adviser was invaluable in keeping them enrolled and on track.

M4Uni students said…

"I am a lot more confident asking for help and being able to put into words what it is that I am actually struggling with, rather than saying 'all of it'"

"I want to make sure I can do everything."

3.3 Discussion of phase three

Students who regularly participated in M4Uni benefited from working with their peers – focusing on the same content with students with similar goals and aspirations. This expectation, that they work collegially as a team, supporting each other, was important to the success of the programme. Each student came into the programme with different strengths and areas for development. As mathematics involves scaffolded learning, students needed to understand one concept before moving on to the next. If students felt responsible for, and contributed to, the learning of others in the 'team', they were more likely to be patient with their peers and persist with the programme. New concepts were embedded through application and practice, and also by discussing them with their peers.

Students sometimes mistake familiarity of a subject with knowledge (Brown, Roediger, & McDaniel, 2014). For example, when a new topic was introduced a student said, "I learned this at school". Whereas, although they recognized the topic, they did not understand the concept, how to apply it, or how it builds on what they already knew. This is an example of poor metacognition – their judgment about what they knew was inaccurate.

Metacognition and self-efficacy are linked to motivation. Initially students may have joined M4Uni because they were extrinsically motivated with short-term goals of passing their assessments. The challenge for the project team was to develop students' intrinsic motivation and strengthen lifelong learning. Over time, we observed that M4Uni improved students' metacognitive skills and their confidence in their ability to learn mathematics. We noticed a change in student motivation and attitudes: they were not satisfied with answers and persisted with learning until they understood concepts. Frequently, at the end of two hours students were reluctant to leave M4Uni until they were satisfied; for example, they could solve a problem without using a calculator or apply a strategy to one more example. Students who started out in the programme with bored expressions and eyes that glazed over when numbers were displayed changed into students who were having fun and working hard to understand mathematics. Students said they kept coming to M4Uni because they wanted to understand; not because they wanted to pass their assessments.

Conceptual learning is challenging, and it can be difficult to engage students. It takes time; it is a longer process to develop conceptual understanding than to fill gaps in procedural skills. Progress was slow at the beginning of the M4Uni programme. Students who attended M4Uni sporadically were disadvantaged as they missed essential learning.

3.4 Recommendations from phase three

Timing and duration are important for achieving the aims of the numeracy development programme. Early intervention is critical - before students experience setbacks and, potentially, disengage from learning. Developing conceptual knowledge takes time and cannot be achieved without a substantial time commitment. Therefore, starting early, continuing throughout the year, and removing barriers or constraints that prevent students from fully engaging with the intervention are key points to consider.

Adhering consistently to the growth-mindset model is essential. This requires deliberate attention to language in all communications, including naming of the programme.

Students are receptive to learning support at different stages – some may not be ready to engage at the start of their university studies. Therefore, it is recommended that a numeracy support programme should be available to students on a regular basis not only in the first semester of their first year. For students who have completed an intensive numeracy programme, on-going support is recommended; for example, consultation with a specialist learning adviser or drop-in maths centre.

The specialist mathematics educator was a key factor in the success of this pilot programme. Although sessions were planned in advance, the SME was flexible and adopted a contingency teaching approach where students' responses to material determined how the session proceeded. (Van de Pol, Volman, & Beishuizen, 2011) SMEs who can unpack mathematics concepts back to basics and develop students' understanding while building students' confidence are a scarce resource. It is recommended that tertiary education organisations recruit and/or develop SMEs and consider succession planning to ensure continuity of support for students.

The Māori student cohort was diverse with some students coming to university from Māori language immersion schools. These students may identify as English as an alternative language. The number in the pilot was too small to generalize; however, it is recommended educators and learning advisers are aware that the numeracy learning development needs of this group may be multifaceted.

Elements in the design and delivery of M4Uni contributed greatly to student engagement and the overall success of the pilot. Active learning engaged students – sessions were very hands-on in contrast to their learning experiences in lectures. Social media examples were used as a vehicle for engagement. A safe environment where learners were comfortable to take risks, voice doubts, and ask questions without rebuke or embarrassment was important, too. To maximize engagement and learning it is recommended that sessions are student-centered and fun, as well as relevant to students' discipline and familiar contexts.

Conclusion

Society's need for quantitative understanding and mathematics knowledge is increasing and graduates need numeracy competency regardless of their degree or career choice. There is widespread concern, however, that many students are not well-prepared coming to university and struggle with quantitative papers. This project piloted an innovative, co-curricular programme that developed students' conceptual mathematics knowledge to build a strong foundation for further learning. This intervention complements existing support offered by TEOs that address gaps in students' procedural knowledge.

The valid and reliable TEC numeracy assessment tool provided data useful for identifying students who would benefit from support. An academic adviser assisted individual students to interpret their numeracy assessment report and prepare a learning plan. If appropriate, students were recruited into the numeracy development programme, M4Uni. A specialist mathematics educator facilitated this programme aimed at improving students' conceptual mathematics knowledge and skills, and addressing maths anxiety, so students would become confident to learn and use mathematics. A purposefully designed learning environment and contextualization of M4Uni to the students' academic programme, added value for students and encouraged engagement in the intervention.

Developing conceptual knowledge takes time; M4Uni was delivered in two-hour, weekly sessions throughout the academic year. The programme was monitored and evaluated to inform continuous improvement and provide evidence of the efficacy of the project.

The project was successful in targeting 'at risk' students and providing mathematics learning support to improve their academic success. Using a growth-mindset model, the intervention boosted students' confidence and addressed affective barriers to student learning. Overcoming their fear of mathematics, improving their numeracy competency and academic success, may have lifelong benefits for students. It is likely students will apply new knowledge and confidence beyond the project completion, as well as beyond university. That is, students are likely to apply their improved conceptual mathematics knowledge and skills to advanced papers, contributing to their academic progression and degree completion; and onto further opportunities for academic specialisation, further educational (e.g. postgraduate) and employment pathways.

Key factors contributing to the success of this pilot included: a cohort of energetic and motivated students; an extremely competent and experienced specialist mathematics educator; and enthusiastic support from staff in the physical education department.

The small number of students participating limited the validity and generalizability of the research evaluating the development programme. However, this project has demonstrated a demand from students for numeracy development support. Once a successful programme such as M4Uni is established in a TEO, word-of-mouth promotion from students and educators would most likely result in an increase the number of participants. Although some aspects of the project are resource intensive, the pilot could be successfully up-scaled for larger student cohorts and adapted to different discipline contexts.

Appendix one:
Numeracy development programme implementation: content and examples.

This table provides a summary of the content addressed in the numeracy development programme, Mathematics for University (M4Uni). Key ideas for each section of content is explained with examples that illustrate: NOW - student knowledge and misconceptions before they started the programme; MOVING FORWARD – content and ideas developed during the programme; and END - the level of knowledge expected when a student has successfully completed the programme with examples from the discipline contexts of physical education (#), business (*), and pharmacy (^).

Breaking Barriers

Key Ideas	Now (Student prior knowledge and misconceptions)	Moving Forward (Content)	End (Applied in context)
Personal mathematical journey Positive attitudes Self-confidence Gender Ethnicity Why do we make errors? Mathematical complexity Mood and inclination Creativity Relevance of numeracy and mathematics to area of study Experience and expertise Quality and clarity of tasks Problem solving techniques Making it fun and enjoyable to break down barriers caused by maths anxiety	• Learner believes that they do not need to grow conceptual knowledge or need any understanding of numeracy constructs for their chosen paper, whereas the conceptual knowledge is implicit • What the learner can do procedurally and what the learner can understand conceptually are not the same • Lecturers expect learners to apply mathematical knowledge and do not have the time to spend teaching what is considered to be 'the basics' • Learners want a 'quick fix' to passing assessments; they want procedural competency at the expense or time spent on conceptual knowledge • Gender stereotypes: learners believe there is a gender imbalance for mathematical ability • Lack of confidence and or lack of positivity • Gaps in learners' mathematical education • Learner is not in the right mood • Learner answers problems based on what they 'imagine' the task is asking (misinterpretation) • Learner feels a sense of failure with mathematics and thinks they *cannot* learn – *the shutters go down* • Learner feels that the work is not relevant to them, their	• Safe learning environment • Inclusive learning environment • Collaborative learning • Peer support • Small group size allowing for catering of individual needs • Breaking down barriers to allow risk-taking • Support structures and academic advice easily accessible • If the task is too difficult for the ability of the learner, misconceptions and errors will be made. Diagnose conceptual knowledge and fill the gaps. • If the task is presented in a way confusing to the learner. Think about the language used and whether the intentions are clear. Consider whether it is translational complexity. • Teach creatively. Broaden the context if necessary to provide relevant learning experiences which engage learners and reduce the risk of errors. Linking to past-experiences will help the learner construct new learning • Educator must know how to teach conceptually not procedurally. • Experience – knowledge can be gained by making mistakes. Remember to FAIL is only the First Assessment In Learning. • General problem-solving strategies such as draw a diagram, make a table, write a list, look for a pattern,	• Being more confident across a range of areas of study • Accept the challenge and take a risk with learning • Change of perspective – from focussing on getting the right answer to focussing on how to get there • Implication for R&D in business • Peer group support enhanced • An ability to question the reasonableness of answers • A positive "can do" attitude where a range of problem solving strategies used to break the task into manageable pieces.

	area of study or their lives outside tertiary study • Learner has limited strategies for problem solving, especially if the task is unfamiliar and they are unable to draw on past-experiences	simplify the problem, act it out, guess and check etc. Effective Pedagogy (NZ Curriculum ➢ Effective teaching ➢ Supportive learning environment ➢ Inclusive learning environment ➢ Reflective thought and action ➢ Relevance of new learning ➢ Shared Learning ➢ Prior learning and experience ➢ Opportunities to learn ➢ Teaching as inquiry ➢ E-learning *www.seniorseconday.tki.org.nz*	

Place Value

Key Ideas	Now (Student prior knowledge and misconceptions)	Moving Forward (Content)	End (Applied in context)
Our *decimal* system of numbers is based on 10, where zero is a place holder. The value of the places increases by a factor of 10 (1, 10, 100, 1000 etc.) as we move up from one place to another and decreases by a factor of 10 as we move down. The pattern “one, ten, hundred” repeats for the thousands (one thousand, ten thousand and hundred thousand) and the millions (one million, ten million and hundred million) giving a repeating pattern of three places. This helps with recognising and reading large numbers (number knowledge) Conceptual knowledge of place value is crucial for part-whole number strategies such as place value addition, subtraction using renaming and place value multiplication and division (using partitioning.) Multiplying and dividing whole numbers and decimals by powers of ten	• Learner gives the number of tens in the whole number 123 as 2 rather than 12 tens; Learner gives the number of tenths in the number 123.4 as 4 rather than 1234 tenths • Learner cannot name large numbers such as “2,034,560” or “102 034 005” • Misconception – when we multiply by 10,100 or 1000 we add zeros to the end of the number e.g. gives 23 x 1000 as 23000 correctly, but gives 2.3 x 1000 is given as 23000 or 2.3000 rather than 2300 • Misconception – when we multiply or divide by powers of ten the decimal point moves, but it is the digits which move place • Learner cannot state the accuracy to which a measurement has been made e.g. 3.450 metres has been measured to the nearest thousandth of a metre (which is the nearest mm). This is particularly relevant in any context where accuracy and error are important	• Knowing the difference between Place Value (PV), Face Value (FV) and Total Value (V) • Knowing the number of tens, hundreds and thousands in any number (Early Additive) • Knowing the number of tenths, hundredths and thousandths in any number (including decimals) • Ordering whole numbers and decimals • Knowing what happens when a whole number or decimal is multiplied or divided by 10, 100 and 1000 (powers of ten) by moving the place value of each digit. This can be extended to introduction of standard form • Naming decimal numbers in tenths, hundredths and thousandths • Writing and saying large and small numbers • Place value addition and subtraction (mental strategies) • Use of place value within the addition algorithm and subtraction by decomposition	Authentic use of place value in context; e.g. • Converting units of measure at home, in the workplace and for course specific needs. #Converting *Length* (running and throwing distances): km, m, cm, mm; #^Converting units of *Mass* (for body mass and BMI calculations): tonnes, kg, g, mg; in ^Understanding capacity (blood, plasma, drug concentration calculations): kL, L, mL. • Extended use of units and exponents when converting units for area and volume e.g. *For making calculations with building and land areas, packing volumes (e.g. m^2 to mm^2; hectares to m^2; m^3 to L; cm^3 into mL or L or kg) • Knowing metric prefixes (tera, giga, mega, kilo, hector, deca, deci, centi, milli, micro, nano, pico) and associated contextual use e.g. terabyte, megawatt, nanometre • Explaining scientific and engineering notation e.g. 12.3 nanometres in *engineering* notation is 12.3nm or 12.3×10^{-9} m whereas in *scientific* notation it is 1.23×10^{-8} • Use of standard form to describe relative sizes of e.g. a red blood cell or a human hair; the speed of light or sound • Using scientific notation on a calculator

Measurement - Units and Mensuration

Key Ideas	Now (Student prior knowledge and misconceptions)	Moving Forward (Content)	End (Applied in context)
Measurement can be divided into several key concepts: units of measure, perimeter, area, volume and capacity, time and temperature. Whilst these concepts can be taught separately it is important to make connections between them and continually make use of materials before imaging and exploration of number and measurement properties. The use of practical measurement tasks is both engaging for the learner and enlightening for the educator as any misconceptions lead to worthwhile discussion and constructive discovery Make measurement **estimates** of length, mass, volume, capacity and time and then check estimates using materials (practical measuring equipment and devices) Convert within and between metric and imperial units. Knowledge of imperial units and ability to convert into metric units is important as many tools and imported goods (especially from USA) use imperial units Calculating perimeters, areas and volumes (including capacity) of familiar shapes, compound shapes and solids, including • Square and Rectangle • Triangle • Parallelogram • Trapezium • Rhombus	Practical Measurement • Learners have difficulty reading measuring devices e.g. rulers and tapes. They may take measurements from 1 (rather than from 0) or struggle to read the division between marks on a linear scale • Learners may not have encountered non-linear scales such as those found on a baking cone, or on electrical devices (ammeter). Equally, they may not have experience of how linear and on-linear relationships may be representing graphically (an understanding of proportionality is required) Units • Learner has limited conceptual Knowledge of place value and cannot convert within units of measure e.g. 3400mm to 3.4m; 5.6L to 5600mL, 0.04kg to 40g or 40000mg • Learner cannot convert between units of measure e.g. expressing 67 litres of water in cubic metres, 67L of water is 67000mL or 67000cm^3 or 0.067m^3 Perimeter and Area • Perimeter is the 'distance around the outside of a shape.' Given a plan of a room learners may add up the given measurements and omit any lengths not stated. • Typically, learners do not attach the appropriate units for length • In a rectangle measuring 2.5cm by 12mm, learners may give the perimeter as 2.5+2.5+12+12 = 29 rather than 25+25+12+12=74mm or 7.4cm i.e. they do not convert lengths when measurements are provided in mixed units • Learner tries to add or multiply all given values on a shape without conceptual knowledge of area Volume • Learner cannot *visualise* one cubic metre. This leads to difficulties when converting to L (1000L) or cm^3 or mL (1000000mL) • Learners cannot reasonably estimate volumes of solids Circles • Learner has limited knowledge of the transcendental number 'pi'	• Choosing the appropriate unit to suit what is being measured • Using the body to make metric and imperial estimates of length e.g. finger width (1cm), thumb (1 inch), forearm (1 foot), stride length (1 metre) • Understanding the concept of tick marks versus gaps on a scale • Practical measuring activities using equipment (e.g. tapes, rulers, scales, cylinders syringes and jugs, stopwatches, thermometers) to measure lengths, mass, volume and capacity, time, temperature • Understand that there is difference between mass (in kg) and weight (in N) • Make connections between perimeter and area. Shapes with the same perimeter do not necessarily have the same area. Area is the number of *square units* needed to cover the amount of space contained within the perimeter (*units of length)* • Paper models to demonstrate that "half of" and "division by 2" are the same (needed for area of triangles) • Visual models/digital manipulatives to demonstrate the relationship between area of a rectangle and that of a triangle, parallelogram, trapezium and composite models • Use of practical equipment (e.g. 12 one metre rulers) to create a cubic metre. Use it to make estimates for the number of L of water or cm^3 blocks it would take to fill. Volume is described as the number of cubic units using to fill a solid object or shape. Use this conceptual knowledge to calculate the volume of a box, a room, a lung, a measuring spoon etc. • Use layering approach to extend concept of area to multiple layers making up volume of a solid • Use of formulae to make calculations for volume of solids such as cube, cuboid (rectangular prism), prisms (including cylinders) with extension to cones and pyramids if context appropriate. • *Additive* number strategies for addition and subtraction	• #Taking measurements of height (in m, using a tape measure marked in m, cm and mm) and body mass (in kg, using electronic scales), to calculate BMI. Use articles and journals to compare BMI with recommended levels for active, sedentary lifestyle • #All aspects of measurement in sport – distances, times, jump height, pace length, heart rate etc. • ^Taking accurate measurements of active ingredients/drug concentrations, fluid dynamics • *Office, land or building area; maximising the volume of packing containers (boxes, cartons, freight containers) for max profit or min waste • #O_2 and CO_2 volumes and lung capacity, or blood flow through cylindrical cross-section blood vessels. Amount of plasma insulin before or after exercise

• Cube and cuboid • Prisms and pyramids This includes aspects of circle measure such as circumference, area of circles and part circles, volume of cylinders. It can be extended to spheres and hemispheres if contextually valid	(π). Typically, they do not make the connection between pi and the ratio of circumference and diameter of a circle. • Learner experiences difficulty with using formulae for e.g. circumference and area of a circle, often because they do not understand the relationship between diameter and radius of a circle • Learners do not use units which makes interpreting equivalence particularly difficult. In map scales 1:50,000 means 1cm is equivalent to 50,000cm, clearly $1 \neq 50000$. Similarly, 4.5kg is equivalent to 4500g but $4.5 \neq 4500$.	• Making use of the equal (=), not equal ($\neq$) and equivalent ($\equiv$) symbols	

Measurement - Time and Temperature

Key Ideas	Now (Student prior knowledge and misconceptions)	Moving Forward (Content)	End (Applied in context)
An instance of time can be named An instance of time can be expressed in 12 or 24-hour notation Time can be measured using a variety of measuring devices to different degrees of accuracy Time cannot be added using the usual decimal system of addition Whole number temperatures can be integers and fraction of a degree and can involve negative rational numbers. This requires an understanding and knowledge of integer number lines	Time and timetables • Learners may not be able to read analogue time, struggling also with fractions of an hour • Learners confuse 12 noon and 12 midnight (specifically use of a.m. and p.m. Learners may not be aware of conventions for 12 and 24hr time e.g. writing 1pm as 13.00pm • When adding 2hrs 50mins to 3hrs50mins, learners get 6hrs rather than 6hrs40mins (learners use decimal addition) • Students write 2hrs and 30 minutes as 2.3hrs not 2.5hrs (again time is treated as a decimal system) • Learners struggle to read and interpret bus and train timetables • Conceptual difficulty with time-zones and date lines • Conceptual difficulty with increasing and decreasing temperatures, especially if temperatures rise or fall across zero degrees. Typically, learners have little concept of different temperature scales (C and F) and cannot convert or estimate between the two	• Use equipment to take measurements of time e.g. laps of a track, reaction times, time to walk or run specific distances. • Calculating elapsed time (length of time between start and stop times) • Make calculations of time in other countries/time zones • Reading a variety of measuring scales for time and temperature • Reading and interpreting bus and train timetables • Calculating flight times crossing time-zones e.g. NZ to Paris via Sydney	• *Knowing trading hours in other countries • ^Calculating times for drug release • #Comparing track times or adding times for multisport events • #Being able to read a variety of measuring devices, watches, stop clocks, multiple time stopwatches, body temperature thermometers

Whole Number

Key Ideas	Now (Student prior knowledge and misconceptions)	Moving Forward (Content)	End (Applied in context)
Conceptual knowledge of the three laws of number (associative,	• Learner does not apply correctly, the laws of	• Use array models to demonstrate the *commutative law* for addition and	• Learners utilise a range of electronic devices for making calculations. All

commutative and distributive) leads onto correct use of order of operations (often known as BEDMAS, BEMA, BOMDAS). Use of these laws to simplify mental computation. Number knowledge consists of number sequencing, place value and basic facts. With whole number, the emphasis is placed on recognising, ordering and sequencing numbers up to one million; addition, subtraction, multiplication and division facts; rules for divisibility; tidy and compatible numbers, factors, primes, multiples and integers Clarifying types of number e.g. real, integer etc. and generating number sequences e.g. squares	number (commutative, associative and distributive). • Learner misuses commutativity law and changes 4-7 to 7-4 rather than -7 + 4. Commutativity does not work with subtraction (unless subtraction is thought of as addition of a negative number) • Typically, the learner will try to use the commutative law to subtraction and division e.g. learner may understand that 4+5=5+4 and 4x5=5x4 but will assume that 4-5=5-4 or that 4÷5=5÷4 • Typically, the learner will work left to right in a problem and will not regroup addition or multiplication to simplify a problem using the associative law e.g. 27+32+3 can be re-grouped as (27+3) + 32 =30+32=62; or 5x17x2 = 17x(5x2) = 17x10 = 170. Learners may (incorrectly) try to use the associative law for subtraction and division • 2+3x4 is given as 20 rather than 14. Learner has not used order of operations • Learner can skip count the 'times tables' but does not know their basic multiplication facts by rote. Learners may not see that 2's, 4's 8's are linked as are 3's, 6's, 12's. • Given a known fact e.g. 5x6=30, learners use repeated addition to progress to 8x6 by adding 30+6+6+6, rather than use of known fact (8x6=48) or distributive law to write 8x6=5x6+3x6 • A number of misconceptions arise with integer arithmetic. Learners believe "2 negatives make a positive" so they will often write (-2) x (-4) = +8 (correctly) but -2 + -4 = +6 (incorrect) • Learner typically confuses squaring with multiplication by 2 e.g. 4 squared is written as 8 • Learner does not transfer knowledge of square numbers to applications requiring area of squares • Learner struggles to write an expression or a word equation which represents a contextual problem where the order is important e.g. Sally gets $36, this is five dollars less than four times Joes money. How much does Joe get? Learners may write 36= 5 - 4 x Joe (or equivalent) rather than 4xJoe-5=36	subtraction, the *associative law* for addition and subtraction and the *distributive law* for addition, subtraction and multiplication. • Using the laws of number to introduce the concept of order of operations • Divisibility rules using materials and imaging and number properties. Begin with rules for divisibility by 10,5 and 2; progress to divisibility by 3,9 then 4,8 and finally by 6. • Basic addition and subtraction family of facts • Basic multiplication and division facts. Using a multiplication grid is useful for discovering which facts need to be supported. Use of commutative law can halve the number unknown facts • Classifying types of number as – natural, whole, integer, real, rational, irrational, transcendental. Many learners are not familiar with this language • Looking at number sequences such as odd, even, multiples, squares, cubes, powers of 10 etc. is useful to introduce at this stage • Contextual problems for order of operations • Integer addition and subtraction should be taught using a number line • Integer multiplication and division can be taught using patterning • Number Strategies for whole numbers (including integer addition and subtraction) should be incorporated throughout this unit. • Advanced additive strategies for whole number (Stage 5) and multiplicative (Stage 6) can be found in Hughes (2007) *Numeracy Professional Development Project - Books 5 and 6.*	disciplines should be encouraged to compare the 'basic' Left to right (LTR) calculators with cell phones, scientific calculators, online calculators and apps, some of which are LTR others use correct order of operations. • Shopping, bill payment or sharing a restaurant bill all require aspects of whole number arithmetic and basic facts • Applying the laws of number underpins the use of algebra, required in most disciplines at some stage • In life outside an academic context - being able to balance a budget and take into account debts and payments in and out of an account • Building confidence with whole number promotes better application of everyday fractions, decimals and percentages • Conceptual knowledge of integers avoids the misuse of rote learned procedures • Using patterning as a method to explore multiplication and division of integers encourages the use of *problem solving strategies,* which can be used time and again when problems are encountered. • In physics courses, the wide variety of formulae cause problems for learners when they have little knowledge of the order of operations and subsequent use of technology (calculators)

Addition and Subtraction Strategies - Whole Number			
Key Ideas	Now (Student prior knowledge and misconceptions)	Moving Forward (Content)	End (Applied in context)
Addition and Subtraction strategies with whole numbers at Numeracy Framework stage 5 and 6. (Stage 7 involves rational numbers such as fractions, decimals and percentages)	• Learner has limited mental strategies for addition and subtraction. Typically, they rely on a single strategy (e.g. place value) or try to use the standard written for (algorithm) in their heads. • Written algorithm is used incorrectly or with limited understanding. It should be used when the numbers are too large to handle mentally. • With money, given $278 in $100 notes, $10 notes and $1 coins, learners would usually know that they would have two $100 notes, seven $10's and eight $1, however, typically they would not be able to say that there could be 27x $10 notes or in the amount $2340 there are 234 x $10 notes or 23 x $100 notes etc. • Learners can add and subtract when given the problem in symbols but cannot decide between add or sub when the problem is in context (words) • Given the problem 124 + 336, learners do not recognise the compatible numbers (4+6 making a ten) • Given the problem 25 + 27, learners do not recognise that the sum is a little more than double 25 • Given the problem 652 – 297, equal addition will allow us to write 655-300=355. Typically, the learner will either have not met this strategy before or will not understand it conceptually (use of a number line is advised) • Learner does not know what to do with the 'carry marks' in place value addition. They have learnt the algorithm procedurally and do not conceptually understand place value. Similarly, subtraction algorithm may have been learnt as a procedure as opposed to decomposition of place value	*From Hughes (2007) Numeracy Professional Development Project, Book 5 – Teaching Addition, Subtraction & Place Value.* Numeracy Stage 4 (Early additive, part-whole thinkers) • Compatible Numbers • Addition and Subtraction in parts • Making numbers up to and over the next ten • Using word stories for add/sub situations • Money - How many $10, $100 notes in amount Numeracy Stage 5 (Advanced additive/Early Multiplicative, part-whole thinkers) • Using materials to model place value addition into the hundreds (saving hundreds) • Jumping the number line • Don't Subtract- Add! • Problems like 23 "and what" = 47 • Tidy numbers (and compensation) • Equal additions • Near doubles • 3 or more additions or subtractions at a time • add/sub with large numbers • Standard written form (algorithm for addition and decomposition algorithm for subtraction) • Using estimation as a check Numeracy Stage 6 (Multiplicative) Addition and Subtraction at stage 6 involves fractions, decimals and percentages (see rational number)	• Learners across all disciplines find these additive strategies useful, especially outside of the university environment (shopping, home). • Improved confidence with number sense and development of a range of different strategies for different types of problem • Making mental calculations quicker and more accessible • #Calculating perimeters, lengths or distances

Multiplication and Division Strategies - Whole Number

Key Ideas	Now (Student prior knowledge and misconceptions)	Moving Forward (Content)	End (Applied in context)
Multiplication and Division strategies with whole numbers at Numeracy Framework stage 6. (Stage 7 involves rational numbers such as fractions, decimals and percentages) Move forward from multiplication by repeated addition and use multiplicative strategies Use of the array model is vital when introducing commutative property for multiplication. Advanced Additive (Multiplication) – knowing facts and using distributive, commutative and associative properties to find unknown facts from the known facts. The multiplicative situation - With multiplication and division, it is important to explore the wide variety of relationships and complex language used. For every multiplicative situation, there is usually a family of two division facts and 2 fraction facts than can be derived. Advanced Additive (Division) – anticipation of sharing and repeated subtraction by reversing multiplication facts. Multiplicative (Multiplication) – Applying the properties of multiplication (distributive, commutative, associative, inverse) to a full range of contexts and whole numbers	• Learner has limited mental strategies for multiplication and Division. Typically, they rely on a single strategy or try to use the standard written for (algorithm) in their heads. • Written algorithm is used incorrectly or with limited understanding. It should be used when the numbers are too large to handle mentally and is worth spending time on. • In division, the learner is unsure which number is being divided, *dividend,* and which number is the *divisor* and when it is appropriate to use remainders (result of sharing) or decimal fraction. • Learner cannot solve division problems such as 624 ÷ 12 (i.e. does not know to express 12 as factors giving 624 ÷ 2 ÷ 2 ÷ 3= 52)	*From Hughes (2007) Numeracy Professional Development Project, Book 6 – Teaching Multiplication and Division* Learners at stage 5 (Advanced Additive/early multiplicative) need a range of strategies including: • Compensation from tidy numbers (rounding and distributive property) • Place Value multiplication • Reversibility and commutativity (changing the order) • Associativity (proportional adjustment) such as doubling and halving, tripling and thirding, halving both numbers, double and double (x4), use of factors • Simplifying division problems using common factors (e.g. 144 divided by 24 as the same as 12 divided by 2 = 6 using common factor of 12 and box model notation) • Interpreting remainders • Standard written form (algorithm) for multiplication and short/long division algorithm Use of factors is expected at stage 6 and can be covered in this section e.g. 27 + 45 = 3x9 + 5x9. Using distributive law gives (3+5) x 9 = 8x9 = 72 Learners at Stage 6 (multiplicative/early proportional) will be using multiplication and division with fractions, decimals and percentages as well as the application of ratio and rates. These can be covered at a different time	• Learners across all disciplines find these additive strategies useful, especially outside of the university environment (shopping, home). Improved confidence with number sense and development of a range of different strategies for different types of problem • Use in part-whole thinking, comparisons, rates, area, volume etc.

Multiplicative (Division) - connecting the properties of multiplication to both forms of division using *inverse* operations across a full range of contexts with whole numbers Learn the multiplication tables as sets of objects, using patterning and known facts to support learning			

Rational Number - Fractions

Key Ideas	Now (Student prior knowledge and misconceptions)	Moving Forward (Content)	End (Applied in context)
The need for fractions stems from division in a *measurement* situation and division in a *sharing* situation Fractions can be a part of a single (*continuous)* object or part of a set of objects (*discrete*) The relevance of the numerator and denomination as part and whole *(part-whole thinking*) is a key idea to improving conceptual knowledge of fractions *Comparison* of fractions to one and one-half and subsequent use of equivalent fractions, allows fractions to be *sequenced* (ordered). Equivalent fractions to any given fraction are infinite and take the same place on the number line. They are simply other names for the same quantity. Contextual problems which involve finding this as a fraction of that (multiplicative relationships, quotient) can be simplifies by changing the order using commutativity Sharing situations can involve representing fractions expressed	• Given the problem that 5 biscuits are shared between 6 people, learner does not conclude that each person gets five-sixths of a biscuit. • Learner may be able to order the fractions: $\frac{3}{9}\ \frac{5}{9}\ \frac{1}{9}\ \frac{2}{9}\ \frac{4}{9}$ (where the denominator is the same) or $\frac{2}{8}\ \frac{2}{5}\ \frac{2}{3}\ \frac{2}{6}\ \frac{2}{2}$ (where the numerators are the same) but not $\frac{7}{8}\ \frac{4}{5}\ \frac{2}{3}\ \frac{3}{4}\ \frac{5}{6}$ or $\frac{2}{6}\ \frac{3}{4}\ \frac{5}{8}\ \frac{4}{10}\ \frac{7}{12}$ (where numerators and denominators vary) • Learner can find a simple unit fraction of an amount (e.g. one quarter of $20) but cannot find three-quarters of $32 or five-sevenths of $42). They do not recognise that $5/7^{th}$'s is equivalent to finding five 'lots of' $1/7^{th}$ • Cannot add, subtract, multiply or divide fractions without a calculator and/or cannot interpret calculator notation e.g. 1⌟2⌟3 as $1\frac{2}{3}$ • Learner has procedural competency for fraction arithmetic (e.g. 'picnic table' models), but lacks conceptual knowledge • Learner cannot simplify fractions such as $\frac{16}{34}$ using equivalent fractions (or division by same factor of numerator and denominator) • Learner cannot estimate their result in a test (this as a fraction of that) as a fraction of 100 (to convert to a percentage).	• Making a fraction wall gives tools for building conceptual knowledge of part and whole, ability to recognise equivalent fractions, comparing and sequencing fractions and making vital connection between addition and subtraction of fractions with the need for equivalent fractions • Modelling equivalent fractions using materials and then imaging and number properties for solving addition and subtraction problems • Box diagrams and arrays are vital for building conceptual knowledge for finding fraction of an amount, finding the part and finding the whole • Modelling multiplication of fractions using layering of fractions (2^{nd} fraction rotated 90 degrees) • Modelling division by a fraction using box models and colours	• Use in shopping e.g. prices after reduction, cooking recipes, time, sharing money, food, equipment • *If the average rate of return increases what happens to the coefficient of variation involves an understanding of the implications of increasing the denominator of a fraction, as does increasing the sample size when calculating margin of error in statistics/quantitative paper. • In test results, learners cannot interpret their mark as a fraction of the total score in any other form. The struggle to compare their marks with a given pass mark or work out what mark they need to get on the next assessment to pass. • Learners often have a procedural competency in handling fractions and can operate on fractions using rote learned techniques, but they freely admit to having little conceptual knowledge, which limits ability to understand or solve contextual problems • In statistics papers, Chebyshev's law requires a conceptual knowledge of fractions to make connections between the percentage of data lying with a specific number of standard deviations from

as mixed, proper and improper fractions. Addition and subtraction with one or more denominator change Multiplication and division of fractions using 'lots of' terminology. When multiplying fractions which are both less than one, the result will be less than either of the fractions Fractions can be *operators* - Contextual problems which involve finding a fraction of an amount Fractions form part of the rational number system and can be converted to different forms such as decimals and percentages Rates and reciprocals (multiplicative thinking) are expressed as fractions Fractions are classified as rational numbers			the mean (standard normal distributions) • ^It is essential that learners are competent users of fractions, decimals and percentages, especially in drug calculations • In quantitative papers including statistics, learners meet fractions e.g. with confidence intervals, test statistics, z scores. Statistics is underpinned by proportional reasoning

Rational Number - Decimals and Percentages

Key Ideas	Now (Student prior knowledge and misconceptions)	Moving Forward (Content)	End (Applied in context)
Fractions can be expressed as different numbers including decimals and percentages There are a variety of different constructs within which decimals and percentages are appropriate-part whole continuous and discrete (split shape or shapes into parts), operator (0.2 x amount), measure (12m divided by 30cm strips or 0.3m), quotient (4 divided by 7), ratio and rates Addition and subtraction of decimals requires conceptual knowledge of place	• Misconception that decimal multiplication has the same properties as whole number multiplication, that multiplying by a decimal always makes things bigger and division makes it smaller • Learner may be able to multiply and divide whole numbers by 10,100 and 1000 but not decimals • Learner knows that 4 x 8 =32 and perhaps 0.8x4 = 4x0.8 = 3.2 but does not make connections with the place value and digits in the problem 0.004x0.08 (=0.00032) • Learner cannot write 0.047 as a decimal fraction (using place values of 10,100,100 and powers of 10 in the denominator to give 47 thousandths or $\frac{47}{1000}$ or as a percentage $\frac{47}{1000}$ = 4.7%)	• Ordering decimals to at least 3 places • Rounding decimals to nearest tenth, hundredth, thousandth, given place value • Rename fractions as decimals • Addition and subtraction of decimal numbers using place value and decimal fractions with denominators or 10,100 and powers of ten • Multiplying and dividing decimals using conceptual knowledge of place value and preservation of digits • Jumping the number line and reversibility (don't subtract – add!) strategies for addition and subtraction of decimals • Using a variety of additive and multiplicative mental strategies for decimal arithmetic • Advanced proportional thinkers see fractions, decimals and percentages as equivalent forms and need to	• A wide range of financial contexts are likely to be used both inside tertiary institution and in daily life experiences e.g. discounts, interest rates, exchange rates, profit and loss, percentage increase and decrease, GST etc. • Business and finance, accounting, economics commerce degrees have many fraction, decimal and percentage problems with contextual applications

value and number partitioning (in parts) Associative and commutative laws of number can make multiplication of decimals easier Mental multiplication and division strategies with decimals e.g. using partitioning and knowledge of factors Terminating decimals can be written as decimal fractions and percentages Recurring decimals can be written as fractions and percentages Percentages have 100 as their denominator Applying the properties of multiplication to a full range of contexts with fractions, decimals and percentages	• Learner cannot write e.g. the fraction $\frac{8}{20}$ as a decimal or percentage or the decimal 0.32 as a fraction or a percentage. • Learner may typically know that 0.90 = 90% and 0.09 = 9% but does not know how to write e.g. 0.0009 as a percentage • Misconception that percentages cannot exceed 100% e.g. Making comparisons between e.g. scores on a test cannot exceed 100%, whereas some yields expressed as percentages can exceed 100% • Learner cannot differentiate between when the *percentage* is unknown (e.g. a student scored 24 out of 36 on a test, what was their percentage score), when the *part* is unknown (e.g. a student scored 75% on a test which was out of 28, what did they score?) and when the *whole* is unknown (e.g. a student got 6 marks on a test which was 12.5%, what was the test out of?)	be able to make conversion between them • Practical and contextual problems using a range of decimals and percentages • Expressing one quantity as a percentage of another • Percentage of an amount • Increase and decrease quantities by a given fraction, decimal or percentage • Percentage change • Financial mathematics (profit, loss, interest, GST etc.)	

Addition and Subtraction Strategies - Rational Number

Key Ideas	Now (Student prior knowledge and misconceptions)	Moving Forward (Content)	End (Applied in context)
Applying addition and subtraction particularly with one and two place decimals involves conceptual knowledge of place value addition, that ten tenths make one whole and ten hundredths make one tenth, vice versa for subtraction Combining ratios can be introduced later but involves the addition of parts and subsequent conversion to fractions and decimals	• Learners cannot add with tenths e.g. 43.8 + 32.7. Typically, they can add 43 and 32 (=75) but are not sure what to do with the .8 and .7 (often giving 0.15 rather than 1.5) writing the answer as 75.15 not 76.5) • Learners cannot subtract with tenths e.g. 63.6 – 42.8. Typically, they subtract 42 from 63 (=21) then they subtract .6 (6 tenths) from .8 (8 tenths) rather than vice versa giving 21.2 rather than 20.8 • Given a series of additions (or subtractions) learners do not typically combine terms (using multiplication) but work left to right	*Numeracy Professional Development Project, Book 5 – Teaching Addition, Subtraction & Place Value (2007)* Stage 6 (Multiplicative) • Tenths arise as a result of division of whole numbers • Adding with decimal fractions using place value • Subtraction with tenths • Using multiplication to add, subtract a series of whole numbers, fractions or decimals • Combining proportions to find the whole (ratios written as fractions of the whole)	• Use of tenths is typically found in applications of measurement (metric units, odometer readings, petrol pumps), percentages and angle. Use of hundredths is most commonly found when dealing with any aspect of currency • *Applications in accounting and financial mathematics

Multiplication and Division Strategies - Rational Number

Key Ideas	Now (Student prior knowledge and misconceptions)	Moving Forward (Content)	End (Applied in context)
Applying multiplication and division particularly with one and two place decimals involves conceptual knowledge of place value, that ten tenths make one whole and ten hundredths make one tenth, vice versa Remainders after a division can be converted into e.g. tenths or hundredths or multiples of ten and subsequently divided Rounding can be used to change decimals into whole numbers but compensation needs to take place	• Learners cannot multiply whole numbers by tenths e.g. 35.3 x 5. Typically, they can find 35 x 5 using place value repeated addition or multiplication but cannot handle the .3 • Learners cannot express 4.5 divided by 3 without a calculator. The know that 3 will divide into the 4.5 once, but do not know how to express the remainder as a decimal i.e. that 4.5÷3= 1.5 i.e. 4.5 ÷3 = 1 and (1+5 tenths ÷3) 4.5 ÷3 = 1 and (15 tenths ÷3) 4.5 ÷3 = 1 and (5 tenths) 4.5 ÷3 = 1.5 • 4.92 x 6 can be rounded to 5x6 (=30) but typically learners are unable to compensate by subtracting 0.08x6 (or 0.48) to give 29.52	*Numeracy Professional Development Project, Books 5 and 6 (for* Stage 6) • Place value multiplication with tenths (3x1.5) • Division with tenths (5.2÷4), “goesinta”/proportional packets idea • Rounding and compensation • Decimal multiplication and division by 10, 100, 1000 • Division strategy using doubling to gain a power of ten Multiplication and division of *fractions* can be covered separately	• Accounting, economics, business and financial applications • Used with currency, measurement etc. • Conversion of units in PHED, Pharmacy, Physics

Algebra

Key Ideas	Now (Student prior knowledge and misconceptions)	Moving Forward (Content)	End (Applied in context)
Using patterning to explore generalisations *Letters* are used in two ways, for *unknowns* and for *givens* (knowing of the meaning of parameter and variable is important). Acceptance of the lack of closure when operating **on** and **with** variables (writing equations to model problems and situations) Understanding of equality. The ‘=’ sign does not mean work out the answer to the problem but is a statement of equality or balance e.g. if 5+6 = 11, then anywhere we see 11 we could replace it with 5+6. Operations on objects versus process.	• Learners may have a knowledge of the laws of number (commutative, associative, distributive and inverse) but may not be able to apply these to algebra • Learners consider the ‘=’ sign to mean ‘work out the answer,’ this means that the concept of equality is not understood. • Learners are confused when we use ‘letters instead of numbers.’ Typically, they cannot comprehend that a pronumeral such as ‘b’ can have different values. • E.g. *Given* that b=5, what is b=2? means that we are ‘given’ the value of b • E.g. a single value as in b+1=6 means the value of b is *unknown* and that b=5 or if b+8=9 means that b=1 • E.g. one or two possible values as in $(a-7)^2 = 0$ gives a=7 or (a+7)(a-7) = 0 where a=7 or -7 • E.g. an infinite number of values as in 2(b+3)=2b+6 • E.g. no possible values as in b+3=b+4 • E.g. no real solution as in $b^2= -16$	• *Parameters* and *variables* • Given’s and unknown’s • Explain meaning of *expression* and *equation* and *formulae* • Meaning of *equality* and *equivalence* • Law of number applied to algebra • Moving learners from *procedural* thinking (process or operational) where they tend to work left to right working out each ‘answer’ as they go versus *object* (structural) thinking where the expression is thought of as a whole object which can be manipulated. • Manipulating algebraic expressions (collecting terms, simplifying, expanding, factorising, substituting) • Rearranging algebraic equations to solve or to change the subject of the equation. Using specific context from individual areas of study is vital here • Solving algebraic equations and inequations	• *Substituting for fixed cost or variable cost to calculate total cost • #Rearranging Distance = velocity x time (d=vt) to find velocity when given distance and time • In physics, where formulae need to be rearranged, variables substituted, expressions combined. • *Exponents can be used to simplify compounding interest rates and formulae can be used to calculate principal or interest on investments. • In many areas of study, the use of formulae (and possible rearrangements) is expected • Expressing situations using linear equations to model limits e.g. the number of text messages sent in a month must be ≤250 • *Understanding the link between globalisation and income inequalities

The meaning of equivalence in algebraic reasoning It is important that correct algebraic statements are made which avoid the use of 'running arithmetic' Correct use of algebraic syntax In algebraic reasoning, algebraic statements need to be thought of as 'objects' (structural thinking) rather than 'processes' (procedural, operational, process thinking) Conceptual knowledge of Commutative, associative, distributive and inverse laws as they apply to NUMBER is essential before learners are able to apply them to algebraic statements. Inverse operations need to be introduced prior to solving equations A range of numbers need to be considered Algebraic manipulation includes simplification, order of operations, brackets, exponents, inverse operations. Additive thinkers need multiplicative thinking. Solving Equations, in-equations and rearranging equations (in a variety of contexts both algebraic and contextual) needs to be modelled using a variety of notations, including box models and number lines.	• Learners have poor knowledge of algebraic convention e.g. commutative c x 2 is written c2 (and often confused with c^2) rather than 2c. Similarly, c x a x b is written cab rather than abc or c x (-a) x b is written c-ab rather than –abc. • Use of '/' and '÷' rather than use of fraction notation makes rearrangement of algebraic formulae more difficult for learners • Making use of the equal (=), not equal (≠) and equivalent (≡) symbols. Learners do not make connections between equivalent statements e.g. given 20 x 7 = 140 then we can say that 19x7=140-7 or 21x7=140+7 • Multiplication sign (×) confused with pronumeral x • Learners tend to use *running arithmetic* e.g. when solving b+4=7, they may know that the first step is to subtract 4 (from both sides) but write b+4=7-4=3. The equal sign now means that b+4 is equal to 3 as opposed to b=3		

Proportional Thinking - Ratios and Rates

Key Ideas	Now (Student prior knowledge and misconceptions)	Moving Forward (Content)	End (Applied in context)
Ratios describe part:part comparisons (and sometimes part:whole comparisons). They can be written in the form a:b meaning there are 'a' equal parts of one object for 'b' equal parts of another. Usually, 'a and 'b' represent parts of the whole and therefore must be of the same 'type' or attribute. Learners need to be part-whole thinkers since ratios describe a multiplicative relationship between two (or more) quantities of the same type. Ratios can be written as *fractions of the whole,* giving *part-whole relationships*. In the ratio "red counters: blue counters", 4:5 means there are 4 red counters for every 5 blue counters. In fractional notation $\frac{4}{9}$ of the whole number of counters are red and $\frac{5}{9}$ of the counters are blue. Ratios can also be written as *fractions of each other*, i.e. to compare parts with parts. In the ratio above there are $\frac{4}{5}$ as many red counters as blue and $\frac{5}{4}$ (or $1\frac{1}{4}$) as many blue counters as red. Converting part-whole relationships to decimals or percentages allows comparisons between the parts (helpful in solving contextual problems.) Rates describe multiplicative relationships between two *different* attributes. Rate problems can be solved by	• Ratios can be simplified (similarly to fractions). Learners who are additive thinkers or have poor basic facts, will struggle to find common factors e.g. learners can simplify 10:20 as 1:2 but cannot simplify 51: 36 • If money is split between person A and person B in the ratio 2:3, learners will typically write that person A get two thirds of the money (as opposed to two-fifths). Their confusion lies in the fact that person A gets $\frac{2}{3}$ as much as person B gets but $\frac{2}{5}$ of the whole amount. • In the *equalising* problem: 3 adult movie tickets cost $25.80, how much do 6 tickets cost? Learners are not able to find the missing measure using a *scaling* strategy- 3 cost $25.80 so 6 costs $51.60 (doubling) • In the *equalising* problem: 3 adult movie tickets cost $25.80, how much do 7 tickets cost? Learners are not able to find the missing measure using a *between* strategy- 3 cost $25.80 so 1 costs $25.80÷3 =$8.60, now 7 costs $8.60x7 =$60.20 • In the *comparison* problem: person A earns $153.00 for 9 hrs work and person B earns $181.50 for 11 hrs work, who has the highest pay rate? Learners are not able to find the best rate using a *between* strategy which uses the 2 measures to create a rate i.e. person A gets $153÷9=$17/hr and person B gets $181.50÷11=$16.50/hr so person A has the higher pay rate. • Learner relies on use of a calculator to solve a rate problem with little understanding of what they are doing. Typically, they cannot solve the problem using mental strategies. • Learner may be able solve equalising or comparison rate problems using a calculator but without conceptual knowledge they cannot apply this knowledge to problems involving time e.g. If it takes 5 men 2 hrs to dig a hole, it will not take 10 men 4 hrs! In this case, the scaling strategy does not work.	• Use ratio to solve problems involving relationships between quantities including simplifying ratios • Use ratios to involve situations involving sharing • Use ratios to make comparisons • Use scaling strategy to solve equalising rate problems • Use between strategy to solve equalising rate problems • Use scaling strategy to solve comparison rate problems • Use between strategy to solve comparison rate problems • Use rates to describe the slope or gradient of graphs • Use rates to solve proportional problems in context e.g. exchange rates • Use rates to solve ratio problems involving time	• ^The use of comparisons is required for rate problems involve drug dosage e.g. a 50kg person requires 10mL of drug. How much should be given to a 72kg person? • #Relating the gradient of a distance against time graph as representation of the rate at which distance is changing over time, in other words speed (or velocity). A steep linear line represents a high rate or speed, a shallow gradient represents a low rate or speed. Similarly, for other kinematic situations. • *The use of rates (e.g. exchange rates) • *The accounting rate of return is relationship between the accounting profit and the amount of the initial investment • In Physics, kinematics requires knowledge of rates of change

equalising or comparing Linear proportional relationships which come from ratios or rates, can be represented using double number lines or as graphs using sets of ordered pairs			

Tables and Graphs

Key Ideas	Now (Student prior knowledge and misconceptions)	Moving Forward (Content)	End (Applied in context)
Evaluate the effectiveness of different types of graph to represent given sets of data Represent data using a variety of charts, tables and graphs Interpret information from tables and graphs, discussing components of the data and making statements based on the data Read within and beyond the data. Use the graphs and data to make predictions, if appropriate Describe and compare graphical features of data	• Learners have developed poor graphing skills, typically, they cannot read the divisions between given values on a set of axes unless there are e.g. ten divisions increasing in one's • Learners draw axes starting at zero. Typically, they do not know how to indicate a break in the graph or a non-zero start to the axes • Learners believe that both axes should 'look the same' and divisions on both axes should increase by the same amount, as opposed to the different axes having different scales. Similarly, learners may change the number of divisions within a single axis (e.g. intervals of 2 and then intervals of 5). This is typically seen when learners change the size of each class interval. The most common is using 0-10 and 11-20 where the first interval has 11 values and the second has 10. • Axes are poorly labelled (often without units if appropriate), title is either missing or weak. • Poor choice of scale, typically learners will draw a graph which is too small to read or the required scale extends beyond the page. • Learners do not know how to use technology to create/alter the parameters of a graph • Learners cannot plot points on a graph -they often confuse the 'x' and 'y' ordinates. • Learners cannot make decisions about whether the data is continuous or discrete. This means that they cannot make decisions about which graph type best fits the data. • Learner believes that correlation means causation	• Discussion of dependent and independent variables should extend to plotting points on a rectangular (Cartesian scale) • Scales should be anchored at zero but can begin at any value provided an axis break is used. The scale should be chosen appropriately with equal divisions. • A good scale could display powers of ten or powers of 10 multiplied by 2 or 5 (if possible). Exceptions could be e.g. scales which use 'months' or quarterly return • Good graphing skills needs to include appropriateness of scale, labelling of axes, use of units, appropriateness of title, legends (keys), size of graph • It is important to know when to read within the graph (interpolate) and when to read beyond (extrapolate) the graph. Extrapolation is not always appropriate and will depend upon the context, the validity and the confidence the learner has in the data. Learners can use graphs to make predictions • Sorting and categorising data? • Reading information from tables and graphs. Use of 'sectioning' will enable the learner to make better interpretations of the data • Use of appropriate context. • Drawing a variety of different types of graph which best fit the data	#Numerous graphs can be used to emphasise particular features for example: Gap in graph – during repolarisation phase in the action potential of heart muscle. Comparison graphs – amount of insulin over time before training and then after training S-shaped curves – use of a percentage scale to show percentage of oxygen saturation of haemoglobin Discrete combined bar graphs – Bars show blood leukocyte count before and after exercise at discrete 30 second intervals Time series – stores of body fat in African women reflecting the times of surplus and famine Scatter graph – energy expenditure against heart rate (direction, strength/goodness of fit/ correlation, slope, context) Picture graph – used to translate a line graph into pictures e.g. the force a weightlifter needs to lift dumbbells at different points in time Kinematic graphs – Velocity, distance, time, acceleration, vertical height of a high jumper Family of curves – shotput distance for various trajectory angles

Statistics - Types of Data and Statistical Measures

Key Ideas	Now (Student prior knowledge and misconceptions)	Moving Forward (Content)	End (Applied in context)
There is a direct relationship between number knowledge and strategies and the learners' performance in statistics Data can be numerical, categorical or ordinal. Measures that describe data are called statistics Graphs and statistics together can give a sense of the 'shape' and distribution of the data Central tendency is a single value or statistic based on a set of data, that represents the middle or the centre of the distribution. Mean, Mode and Median are all measures of central tendency. Whilst they give us a measure of how centred the data is, they do not give information about the spread/ shape/ distribution of the data. Knowing when it is appropriate to use the mean, median and mode in different contexts Data sets can have no-mode or be multi-modal. Measures of spread (range, IQR, variance, standard deviation) are used to describe the *variability* of a sample of data or of a population. They give an idea of how good the measure of centre is in describing the data set as wells as providing insight into the distribution of the data.	• When asked to identify the middle (median) of the set of data {12,3,5,11,9}, the learner may not put the data in order first giving the median as 5 not 9. • Learners may be procedurally competent in calculating the mean and median, but they do not know which average is best to use in a given situation. E.g. In a small company the following wages are paid out to employees {$28000, $32000, $30000, $29000, $31000, $30000, $225000}. The median wage ($30k) and the mean wage is $57857. For a new employee, the median wage is more realistic than the mean wage. • When asked to calculate mean room rental cost per week given the data set {$125, $80, $140, $100}, learners calculate the mean as $370 not $111.25. This indicates that they have not understood order of operations (i.e. 125+80+140+100÷4=$370 as opposed to (125+80+140+100) ÷4=$111.25 • In the data set {1,2,3,4} learners will say there is no mode, but for {1,1,2,2,3,3,4,4} leaners give the mode as 1,2,3 and 4 (or say multimodal) rather than no mode. • Learners may have a superficial procedural knowledge of range (smallest to biggest) but not give range as a single value statistic. Similarly, they may know that the interquartile range is the range of the middle 50% of data but have no concept of what this means or why it is useful to us. • When given two sets of marks in a test, stream A gained mean=56% and s.d.=20% whereas stream B gained mean 55% and s.d. = 5%. Learners are not able to compare the different streams using statistical language demonstrating knowledge of the variability of the two streams, consistency of marks, implications for teaching and learning, consistency etc. • When asked to comparing different variables e.g.	• The median tells us the value below and above which 50% of the data lies. • The median does not have to be a value in the data set • The median is not affected by extreme values unlike the mean. • Numeric data has meaning as a measurement (continuous data) e.g. a person's height or it represents something counted (discrete data) e.g. no. of bonus bonds a person has • Categorical data represents characteristics e.g. gender. They are usually words (male, female) but can be numerical (1=male, 2=female) where the numbers have no mathematical meaning • Ordinal data can be either numeric or categorical e.g. 1=low to 5=high. Here 1-5 are treated as categories but can also be manipulated mathematically as the numbers have meaning. • The average is a number which describes a set of numbers. The average gives a measure of how 'centred' the data is. • Quartiles tell us about the spread of each quarter of the data, they are less affected by outliers or a skewed data set • There can be more than one mode, 2 modes mean bi-modal, more than 2 modes mean multi-modal. It is possible to have no mode if every piece of data occurs the same number of times. • Spread includes range (for the whole set of data), interquartile range (the spread of the middle 50% of the data), variance and standard deviation (a measure of how far each data point is from the mean value.) • In any numerical or ordinal distribution, we can describe the spread of the data using variance or s.d. provided we are clear which we are using. Variance has squared units compared to s.d.	• In quantitative courses many aspects of statistics may be regarded pre-requisite knowledge • ^It is necessary to interpret pharmaceutical findings and read statistical reports • *Financial reports, data mining and statistical literacy are imperative • *Range is useful when looking at critically high or low values beyond which the variable of interest must not cross

<table>
<tr><td>Standard deviation (s.d.) is widely misunderstood. In many schools, use of graphical calculators have replaced the need to calculate s.d. manually which means there is no conceptual development</td><td>heights of people and price of jackets, learners may calculate the ranges for each e.g. range for height = 45cm; range for jacket price = $160 and try to compare them. They cannot really be compared as the variables are different. However, the variances or s.d.'s can be compared as they give a measurement of the deviation of each value from its central value, in other words a measure of dispersion.</td><td></td><td></td></tr>
<tr><td colspan="4">Statistical Graphs</td></tr>
<tr><td>Key Ideas</td><td>Now
(Student prior knowledge and misconceptions)</td><td>Moving Forward
(Content)</td><td>End
(Applied in context)</td></tr>
<tr><td>A question driven inquiry is when we pose a question and use the statistical inquiry cycle to gather analyse and answer the question

A data driven inquiry is when we are given the data and we look for patterns and trends and display the data graphically. Only then are questions formed.

Understanding of the word most in statistics

By analysing data, we are 'telling a story'

From TEC (2008) "Teaching adults to reason statistically":

Can read the data from a graph
This information may be lifted from the axes, individual points, lines or bars etc.

Can read between the data to interpret information on the graph

Can read beyond the data to make predictions

Can read behind the data to connect meaning and context

Statistical graphs need to be connected with everyday life</td><td>When asked to make statement about a pie graph showing 100 adults favourite colour, learners will typically use the word 'most' to describe the section of the graph with the highest frequency. This may not accurately reflect the numbers involved. E.g. If the pie shows blue (21%), green (15%), yellow (15%), orange (20%), purple (19%), Red (10%) – the colour gaining the highest frequency of likes is blue, but 21% of the adults (21) is not most of them.
In a box plot, learners believe that if one side of the box is longer than the other then it must contain more data. Each part of the box contains exactly 25%. The width or length of the box gives an indication of how widely spread the data is.
Typically, learners will draw graphs without appropriate axes, legends, titles etc. (see Graphing)
When asked the difference between a bar graph and a histogram, learners will often say that the only difference is that bar graphs have 'gaps' between the bars. There is no consideration of type of data, class interval, frequency density etc.
Belief that correlation means causality
Belief that all data sets are appropriate and valid. Typically, learners do not question the validity of the data they are using.</td><td>Use of a percentage strip graph to model percentage pie charts.
Understanding that for pie graphs, the size of the 'slice' is a proportion of the whole circle (for angles) or a proportion of 100 (for percentages). Proportional thinking is again developed here.
Comparison between bar graphs and Histograms
Use of charts to compare bi-variate data e.g. scatter graph
Use of charts to compare multivariate data e.g. box and whisker graphs
Use of context to display different statistical graphs. Learners to be given the opportunity to read information from the graph, interpret scales, use patterns in the data to make estimates and predictions within and beyond the data
Discussion of correlation and causality. Causation means that one particular event causes the occurrence of the other event. Correlation between variables does not necessarily mean that changes in one variable is the cause of any changes in the other e.g. as the rate of ice blocks consumed increases the rate of death by drowning also increases. The two events may appear to correlate but eating ice blocks does not cause death by drowning.
Learners should use online tools available through the tertiary institution to access peer reviewed scholarly statistical articles in their subject area
Learners should access data sets from reliable sources e.g. Statistics NZ</td><td>*Statistical graphs are used widely in all aspects of business, finance, economics and accounting
#Statistical methods are used to analyse physical activity. Graphs could include multiple line graphs to represent physical activity levels in adults and adolescents by age and gender; or life expectancy gains from physical activity
Graphs to show projected change in employment in different sectors would be typical in many areas of study in tertiary education
^Prevalence of a particular type of antibody amongst drug users
^Prescription drug expenditure across several years</td></tr>
</table>

Use of a variety of statistical graphs related to context of individual needs of learners Data Literacy		• Use of Tertiary modules available through the library system are a good way of ensuring learners have a reasonable level of statistical literacy	
Probability			
Key Ideas	Now (Student prior knowledge and misconceptions)	Moving Forward (Content)	End (Applied in context)
In equally likely outcomes, what has already occurred (in the past) does not affect the future. In *experiments* involving equally likely outcomes, there may not be exactly the number of outcomes we expect. This links probability to the idea of long-run relative frequency, where the proportion of successes varies but with each repetition, this proportion will vary less and this empirical probability will approach the theoretical probability of success. The exact probability will always exist throughout. Exact probabilities are unlikely in the real world. We can get an estimate by finding the proportion from the number of trials. The importance of using the correct language of probability The concept of fairness Learners need multiplicative thinking in order to convert probabilities between fractions, decimals and percentages. Moving learners from multiplicative to advanced proportional stages of the numeracy framework is key	When given 26 letters in a bag and asked to create a 3-letter word (with replacement), learner will say that aaa is less likely than tpa. *Representativeness* – selections without patterns seem more random than selections with patterns. However, if trials are independent and equally likely then the probabilities will also be equally (i.e. both have a chance of 0.00006) When combining probabilities, learners do not recognise that a mistake has been made when the probability exceeds 1. Learners are unable to express probabilities as fractions, decimals, percentages. After having 2 boys in a family, learners believe they are more likely to have a girl next. This emphasises the misunderstanding of *independence* with a belief that parents are surely “due to have a girl next if they have already had 2 boys” - In independent events the outcome of one event does not affect the outcome of the other. Learners confuse simple events (e.g. toss a head on a coin) and compound events (get three heads in a row). Similarly, they will also confuse the result of an experiment with the true theoretical probability. E.g. if in a hospital, there are 100 male babies born in 300 days then the probability of having a boy is not one third, it is still 0.5 (50% or ½). Learners confuse sample size and sample space. If the sample size is small (a hospital looks at male births over 30 days not 300 days) then it is more likely that the proportion of male births will deviate more. The sample space of births consists of {Male and Female} Learners do not have an organised strategy for listing outcomes and will list them as	Compare equally likely events with events which are not equally likely. It is important to use a variety of materials including cards, dice, spinners, counters, balls, digital manipulatives to introduce the concept of probability. Probability experiments using a large number of trials. By recording the relative frequency after sets of e.g. 10 trials and graphing the relative frequency, learners will be able to see how the graph settles. Use of digital simulations will allow users to explore tens of thousands of trials. Compare expected probability and experimental probability. Introduction of appropriate language such as relative frequency, risk, uncertainty, fairness, sampling, sample space, events, experiments, outcomes, conditional, independent, mutually exclusive, complementary etc. Representing probabilities using a variety of methods including Fractions, decimals, percentages Venn diagrams, two-way tables, trees Knowledge of sample space will help with conditional probability as it uses a reduced sample space. Two-way tables are common. Risk versus relative risk The Normal Distribution requires proportional thinking including an understanding that probabilities sum to 1; If P(A)=0.7 then P(A’) =1-0.7 or 0.3; Rounding up to 4dp; converting decimals to percentages; percentage of an amount i.e. proportional thinking Application of the margin of error requires knowledge of the order of operations and conceptual knowledge of what happens to fractions when the denominator increases or decreases. This leads to better explanations of	Many tertiary courses require knowledge of and the ability to use and apply the normal distribution. *Using probabilities expressed as decimals, fractions and percentages to compare risk and relative risk. *Probability distributions are used frequently in risk management, e.g. when making decisions based on historical gains or losses from investments

	they think of them rather than developing strategies - tabular methods, Venn diagrams and tree diagrams are all appropriate. This relates back to the importance of having a range of problem solving strategies. Often, learners have been taught to use a graphical calculator in school explicitly, to calculate probabilities from a normal distribution. Typically, they can use the calculator to solve routine problems but have no understanding of how to apply this to contextual problems or how to find probabilities without a calculator. Whilst this concept is beyond the scope of 'numeracy' the ideas underpinning the normal distribution If told that a sample size increases, leaners cannot state the effect on standard error, margin of error, confidence interval etc. This is because they do not have conceptual knowledge of denominators of a fraction or concept of division by a fraction	the effect of sample size and confidence intervals.	

Technology

Key Ideas	Now (Student prior knowledge and misconceptions)	Moving Forward (Content)	End (Applied in context)
Knowing the difference between Left to Right (LTR) and Scientific Calculators which use (BEDMAS) Explore a variety of tools for performing computations • Online tools • Online help sites • Wolfram alpha • Spreadsheets • Graphical calculator	• Basic calculators often work left to right (LTR). Given the problem 2+3x4, learners will get the result 20 (2+3=5 and 5x4=20). Scientific calculators apply the rules associated with BEDMAS – i.e. given the problem 2+3x4, learners will calculate the result as 14 (2+(12) =14), with or without their own knowledge of BEDMAS. It may not be clear whether the learner understands BEDMAS is technology is used. • Online calculators may be used in either basic, scientific, engineering or programmer mode. Learners often do not realise that they can change between displays. • Online calculators, often used in university 'labs' have a paper tape display. This is where each step of a calculation or each entry is made on a separate line or tape, thus keeping a running record of all calculations made. Advantages include the learners' ability to review previous calculations for errors. These 'tapes' can be named and saved.	• Identifying the different types of computational device • Knowing when it is appropriate to use technology • Using estimation to recognise when calculations have been entered incorrectly (e.g. brackets missing) • Using technology as a tool to perform multiple similar calculations • Interpreting the display e.g. 2E2 means $2x10^2$ or 200 or multiple ways of displaying fractions • Use of device memory functions to store values to be used later or to store given parameters	• Confidence to use a variety of devices to perform computations where mental strategies are too difficult • Adaptability • Use devices at home and in university setting e.g. labs, tutorials, lectures • #*Use of spreadsheets to simplify the need for performing multiple calculations. Changing the parameters to allow swift re-calculation e.g. effect of a change of rate of interest on investments • In Physics, knowing how to use a calculator in problems involving trigonometry

	• With online mathematical help environments such as Wolfram Alpha, learners can pose a question and get an immediate result, with little or no understanding. They are then unable to apply knowledge in assessments and examinations, by which time it is too late. • The use of graphing calculators (as a tool) allows the learner to model multiple situations very quickly (e.g. graphically) or statistically to aid understanding, making them a great resource, but they are of limited use in a university setting requiring conceptual understanding without the use of a calculator and should be used to demonstrate situations that would otherwise take a long time and detract from the core idea.		

List of abbreviations

BEDMAS	An acronym for an order of operations in algebra
BPhEd	Bachelor of Physical Education
LTR	Left to right
M4Uni	Mathematics for university – the project's intensive learning development programme
NCEA	National Certificate of Educational Achievement
PASS	Peer assisted study sessions
s.d.	standard deviation
SME	Specialist mathematics educator
TEC	Tertiary Education Commission
TEO	Tertiary education organisation
UE	University entrance

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