Central Regional Hub-funded project

Project Report



Using LNAAT data to improve the teaching, resources and achievement in numeracy education

Gary Sharpe and Hannah Hughson





Project undertaken by: Western Institute of Technology at Taranaki (WITT)

Project report prepared by Gary Sharpe and Hannah Hughson

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List of Acronyms / Abbreviations

	Additive Chrotogiae
AS	Additive Strategies
ЕРК	New Zealand European Pākehā
ITF	Industry Training Federation
Learner	Learner who has completed a numeracy assessment
LNAAT	Literacy Numeracy Adult Assessment Tool
Μ	Measurement
MS	Multiplicative Strategies
NF	Number Facts
NS	Number Sequence
NZCER	New Zealand Council Education Research
PRS	Proportional Reasoning
PV	Place Value
RT	Research Team
STP	Secondary Tertiary Pathway
TEC	Tertiary Education Commission
TEO	Tertiary Education Organisations
WITT	Western Institute Technology Taranaki
WPL	Workplace Literacy
YG	Youth Guarantee

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- Industry Training Federation (ITF), Skills Highway Nicky Murray for the work her team actioned to assist the Research Team with employer data.

Research Team

Primary Researcher Gary Sharpe



I have been involved in Tertiary Education since 1982, beginning a journey paralleled by a common number of tradespeople who at a weak moment, agree to tutor a trades-based class and the die is cast. My poor reading skills meant most of my school focus was in numeracy, which is now my primary role, *Learning Skills Tutor, Mathematics* at Western Institute of Technology at Taranaki (WITT). On a daily basis I work with individuals, small study groups, classes and teaching staff in numeracy from foundation to diploma level. In this role I became curious to investigate if the day to day questions I was seeing were related in any way to results learners obtain from the *Literacy Numeracy for Adults Assessment Tool (LNAAT)* numeracy assessments. The research project grew from that seed.

Research Assistant Hannah Hughson



I have recently completed three years study at Western Institute of Technology at Taranaki (WITT), gaining qualifications in Business Administration and Technology. My journey to this point has been punctuated with hassles, problems and frustration to the point where I now understand Dyscalculia from a personal point of view. The struggle has been immense and will probably continue to be, so when Gary asked if I could help with some numeracy research a lot of eyebrows, including mine, were raised. However, the skills from my qualification and my stubbornness in not giving up have enabled us to trawl endless amounts of data, sift, filter, debate, argue its meaning and stay sane within all this (I think).

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The Assessment Tool – Literacy Numeracy Tool (TEC, 2008a & 2008b)

Executive Summary

The Tertiary Education Commission (TEC) of New Zealand introduced literacy and numeracy assessment (the Literacy and Numeracy Adult Assessment Tool – LNAAT) requirements into the tertiary education sector to improve the literacy and numeracy levels of New Zealanders (TEC, 2008). The Western Institute of Technology at Taranaki (WITT) has been implementing these assessments since 2008. The Numeracy Tutor observed students requesting tutorial support on similar mathematical issues. This led to an investigation into the LNAAT numeracy assessment results.

The aim of the research was to identify the numeracy deficit of tertiary learners in the classroom and in employment. This project used a mixed methods approach. Quantitative methods were used to analyse the Tertiary Education Commission numeracy assessment data from the Literacy and Numeracy Adult Assessment Tool for 2016 and 2017. The analysis separates the data into the seven Tertiary Education Commission learning progressions and then filters the data into the first five Steps of learner achievement from the result of their numeracy assessment. Then it includes breakdowns of age, gender and ethnicity at each Step achievement. Quantitative and qualitative methods were used to analyse data from employers provided by Skills Highway New Zealand.

There were 176696 learner assessments analysed in this project with a 10% increase in assessments from 2016 to 2017. The biggest changes were the increase in the numbers of Under-18 year old learners. In each Step, there was at least 100% increase in numbers in this age group. This may be explained by the Tertiary Education Commission's development of programmes which engaged secondary learners with tertiary education providers, for example the Secondary Tertiary Pathways, "3 + 2" and Dual Pathways.

Although there is only a slight decrease in Māori representation progressing from Step 1 to Step 5, there is a significant decrease for Pasifika and significant increase for New Zealand European Pākehā. A quarter of learners at Step 1 and Step 2 do not have English as their first language giving rise to the question of the impact of literacy on numeracy skills. However, just less than a quarter of learners at Step 5 and Step 6 also do not have English as their first language and yet achieve good numeracy outcomes. Over half of the learners at Step 1 are Pasifika who are also four times more represented at Steps 2 to Step 5 than Māori and New Zealand European Pākehā.

Proportional Reasoning is identified as the progression of highest error from this research, therefore has the highest numeracy skill deficit for learners. The report also highlights high error rates in all six remaining numeracy progressions. Basic numeracy deficits are higher in the 13 to 20 year age groups. This is significant because this cohort of learners was most recently exposed to formal maths and numeracy education at school. The results suggest poor understanding of base numeracy from the primary and secondary education sectors.

The length of time taken to achieve correct answers dispels the myth that 'to be good at maths, you must be fast at maths'. Those learners who achieved more highly took longer to calculate the answers. Learners therefore require time to successfully undertake numeracy exercises.

The high number of 'no responses' at Step 1 and Step 2 raises questions on whether this is due to a lack of numeracy skills, or, reduced literacy skills required to achieve numeracy.

Numeracy assessment error rates ranged from a mean of 63.5% in Step 1 down to 33.5% in Step 5. The Proportional Reasoning progression is identified as having the highest rate of learner error from Step Two to Step Five except Step 1 where it is second to Multiplicative Strategies by only 0.5%. It also has the highest mean score across all the Steps at 54%. However, significant error rates were also found in other progressions. Approximately half of learners had errors in Multiplicative Strategies, Measurement and Place Value. Slightly less than half had errors in Additive Strategies, Number Facts and Number Sequence.

Proportional reasoning skills has been identified as having the greatest number of learner errors as well as the skills that employers have observed having the biggest entry-worker skill gap.

A host of resources are available to assist learners to find solutions to numeric problems, however in great many cases these resources are designed for the solution of specific problems. A dedicated, stepby-step, skill building resource that progresses the learner through proportional reasoning to a skill level that suits today's workplace has not been identified. The platform for this work, Pathways Awarua, is currently available, is closely linked to LNAAAT, and has a proven engagement history with tertiary learners.

Recommendations include:

- The development of a specific staircase Proportional Reasoning module in Pathways Awarua.
- Professional development for tutors in presentation of Proportional Reasoning delivery material.
- The availability of a 'read to me' option for numeracy assessments in the LNAAT

Further research using the reading data from LNAAT to link both literacy and numeracy achievements is required.

Section 1: Background

Introduction

The Tertiary Education Commission (TEC) of New Zealand introduced literacy and numeracy assessment (the Literacy and Numeracy Adult Assessment Tool – LNAAT) requirements into the tertiary education sector to improve the literacy and numeracy levels of New Zealanders (TEC, 2008). The Western Institute of Technology at Taranaki (WITT) has been implementing these assessments since 2008. The Mathematics Tutor observed students, individually or in study groups and learning cohorts asking for tutorial support on similar mathematical issues found in the Numeracy Assessment Tool. This prompted the Tutor's curiosity regarding a possible relationship between these students' requests for mathematical assistance and the fields of error within their numeracy assessments.

Context

The LNAAT is used by Tertiary Education Providers, some secondary schools and employers for the purposes of gaining information on the skill levels of learners or employees. It is a predominantly online adaptive tool and is primarily designed to provide robust and reliable information to educators on the reading, writing, and numeracy and vocabulary skills of adults. For educators, this data enables them to tailor delivery material at a level appropriate to the learner skill level, and to gauge improvements in these skills as training progresses (TEC, 2017). The LNAAT also allows learners to track their own progress over time, and it enables educators and organisations to learn from the progress made by groups of learners or cohorts of learners (TEC, 2017).

LNAAT Assessment Tool data enables TEC to understand the impact of national literacy and numeracy policies as well as the performance of Tertiary Education Organisations (TEOs). TEC shared Assessment Tool data with the sector in 2016 to increase transparency and help improve literacy and numeracy performance. The TEC are also using LNAAT to understand the trends in learner progress and to develop benchmarks for statistically significant gains achieved by learners. Full and effective Assessment Tool usage by TEOs and educators is critical to informing future policy and funding decisions in 2016 and beyond. Accurate and comprehensive data from TEOs will ensure these decisions are robust and evidence-based. The LNAAT can be found in Appendix One.

The LNAAT tool comprises a set of learning progressions, a continuum representing a significant learning development. These progressions have been in place since 2008 and they show what adult learners know and can do at successive points as they develop their skills in literacy and numeracy. The Learning Progressions are available to download from the Ako Aotearoa website on the learning progressions page. See the link for further information https://ako.ac.nz/alnacc/. The TEC (2008a) set out the seven Numeracy Learning Progressions of LNAAT and can be found in Appendix Two.

Literature Review

The literature suggests that a numeracy deficit is more of a barrier to employment and employment progress than literacy deficit (Bynner & Parsons, 2005). Participants in this study left school early, frequently without qualifications, and had more difficulty in getting and maintaining full-time employment. The jobs entered were generally low grade with limited training opportunities and poor pay and prospects. Both men and women lacking numeracy skills, compared with those lacking literacy skills, were more likely in their early careers to have been out of the labour market. They were more likely to be engaged in low grade work, in unskilled manual jobs without training and with low pay. Only for promotion were there signs of a more important role for literacy, but this was restricted to men (Bynner & Parsons, 2005).

The majority of female cohort members in this study, were not in full-time employment by age 37. Women with numeracy difficulties appeared especially vulnerable to exclusion from the clerical and sales jobs to which they aspired (Bynner & Parsons, 2005). Men's problems were less clearly differentiated between occupations. The numeracy problem begins early in disadvantaged family circumstances which carry over into problems in keeping up at school.

Teachers had very limited success in identifying incipient numeracy problems in Bynner & Parsons (2005) study. Large numbers of children, who later, as adults, had numeracy problems, did not have their problems recognised while they were at school. It was more likely that problems with literacy were recognised. A strong understanding of basic numeracy is a key requisite for progressive learning (Chinn, 2013).

The relationship between language (words) and numeracy (numbers) has been debated over time; can one exist without the other? The acquisition, teaching, and learning of language and numeracy have often been treated as two separate areas of inquiry and practice (Gal, 1995 as cited in Neil, 2001). However, in everyday life, we experience numeracy, literacy and language as integrated rather than separate discrete subjects. Wooley (2013) notes that "it is difficult to think of a situation involving numeracy that does not also include aspects of literacy and language" (p.76). Yet in formal education, numeracy and language classes are often timetabled independently, suggesting the topics are taught in isolation. However, everyday experiences do not have this distinction. For example, understanding a bus timetable or doing grocery shopping is almost impossible without having a combination of word and number skills.

Conversations with people in a variety of vocations will involve language specific to their vocation; for example, the language of hairdressing, medicine, engineering or accounting incorporate words and meanings of words that are specific to each vocation. Numeracy is not exempt from this vocation specific language, and therefore links and patterns between numeracy and language are unavoidable.

Even when situations do not require numerical calculations, they may involve expressing an opinion based on interpreting statistical information, or decisions involving ideas of chance and uncertainty. Other tasks will involve numbers or quantitative statements embedded in text, e.g. forms, schedules, manuals, technical and financial documents, statistics in the media. The integration of these skill areas underlies the design of various large-scale surveys of numeracy and mathematical skills such as the International Adult Literacy Survey (IALS) and the design of some quantitative literacy tasks in the Trends in International Mathematics and Science Study (TIMSS) and Programme for International Student Assessment (PISA) surveys. Numeracy has its own language that uses 'very specialised vocabulary' (Newmarch & Part, 2007, as cited in Learning Wales, 2016, p.19). An example can be found in Table 1.

Number	Shape	Data	Using number
capacity	Centre	column	calculate
centimetres	Circle	chart	convert
compound	Cuboid	cumulative	consecutive

Table 1: Numeracy Vocabulary (Henderson, 1998, as cited in Learning Wales, 2016, p.109)

Some words can also have particular meanings in a numeracy context which are different from the way they are used in other everyday situations, e.g. difference, sum, average, similar, bar, table, prime.

Strong adult literacy numeracy skills deliver significant economic and social benefits for the individual, for employers and ultimately for New Zealand. Literacy and numeracy is a priority in the Tertiary Education Strategy 2014 – 2019 (TES, 2014), particularly raising the skills of learners undertaking qualifications at levels 1 - 3 in the New Zealand Qualifications Authority

(NZQA) framework. Since 2006, the TEC have engaged in a systemic approach to building the capability of the tertiary sector to strengthen the literacy and numeracy of learners at entry-level tertiary education. TEC have invested in both the development of a high quality national infrastructure to support educators and directly in delivery to learners through a range of funds and grants (Ministry of Business, Innovation and Employment [MBIE], 2014)

The Research Aim

The aim of the research was to identify the numeracy deficit of tertiary learners in the classroom and in employment.

Research questions included:

- What type of questions produces the highest error rate within the LNAAT seven numeracy progressions?
- Is the progression the same regardless of the Step students achieve?
- What is the impact of demographic data such as age, gender and ethnicity on the numeracy results?
- Do the deficits identified in this study align with the deficits identified by employers?

Section 2: Methodology

Research Approach

This project used a mixed methods approach, i.e. integrating quantitative and qualitative research. Quantitative research describes a systematic, empirical, structured way of collecting and analysing data from different sources, involving the use of statistical, computational and mathematical tools to produce numerical data and generalise the findings across groups of people or explain a particular phenomenon. Qualitative research gathers non-numerical data and refers to the meanings, beliefs, characteristics, views, and description of things and not to their counts or measures.

Data Collection

Three sets of secondary data were used in this project.

- 1. The TEC provided the complete 2016 and 2017 LNAAT numeracy data sets for learner assessments. There were 3.6 million student responses to questions in the LNAAT numeracy assessments with the Step 6 learners responses excluded. This included numeracy assessments undertaken by:
 - Workplace Literacy (WPL) groups
 - Intensive Literacy Numeracy learners
 - Secondary Tertiary Pathways (STP)
 - Learners enrolled with TEOs
- 2. The New Zealand Council of Educational Research (NZCER) provided the complete numeracy question bank. Progressions and the difficulty level for each question were included in the data set.
- 3. The Industry Training Federation, Skills Highway, a TEC group established to help employers learn how workplace literacy and numeracy training can help them improve their business. It provided employer information on numeracy skills required in the workplace. The survey can be found in Appendix Three. Data was provided on:
 - Entry-level skills: adding and subtracting more than single numbers, multiplying and dividing more than single numbers, working with ratios, estimating by counting in 2's, 5's and 10's, understanding place value, reading graphs
 - Other key skills

Ethical Consideration

The data used in this project comprised entirely of secondary data that did not identify people. The project was therefore considered low-risk and received approval by the Western Institute of Technology at Taranaki. The project did not:

- Have the potential to cause physical or psychological stress
- Involve any deception
- Engage in any illegal activity

Method

Learners achieving Step Six were excluded from the TEC LNAAT numeracy cohort as this group exceeds the Tertiary Education Commission Step Five thresholds for numeracy re-assessments. The remaining 176,696 learner data sets were used for analysis.

The LNAAT question bank was sorted by progression. The questions have a difficulty value of between -5.043 (easiest) to 5.094 (hardest). These values have been assigned by the psychometricians at NZCER as a reference and align with the Step each question is targeted at.

A filtering system was designed to identify sub-data sets and included:

- The seven numeracy progressions: Additive Strategies, Measurement, Multiplicative Strategies, Number Facts, Number Sequence, Proportional Reasoning and Place Value, (as shown in the list of Progression Steps in Appendix Two).
- Demographic data of age (TEC age groupings: Under-18, 18-19, 20-24, 25-39 and Over-40), gender (Male / Female) and ethnicity (first option selected).

One student identified their age at 13 years. Although an outlier in the data, it remained within the data ensuring the inclusion of STP learners; these are learners currently in secondary education but with a component of their study undertaken in the tertiary sector. Any student identified as aged less than 13 years was not included.

The 2016 and 2017 data were analysed separately except where numbers were low. In that case, the two years were combined such as ethnicity and English for speakers of other languages' (ESOL) data.

The time learners engaged with the numeracy assessment was calculated. After discussions with NZCER, a lower and upper outlier of four minutes (00.04.00) and four hours (04.00.00) respectively were used. The time boundary calculated for the learner to be able to reach a considered conclusion to a minimum of fifteen question between four minutes and the four-hour maximum was deemed a reasonable time frame to enable the learner to disengage and reengage to complete the assessment.

There was no separation of Youth Guarantee (YG) learners from non-Youth Guarantee, as the raw data from NZCER did not support distinctions for this group. They are included in the Under-18, 18-19 and 20-24 years categories as a YG learner could enrol toward the end of their nineteenth year and remain on the YG scheme into their twenties.

Every response to a numeracy question given by a learner was included in the analysis.

Data analysis

Descriptive statistical methods, specifically percentages and means were used to analyse the data sets. Although significance testing is not always relevant for a whole population (although this population did have the Level 6 learners excluded), some of the data was compared for the level of significance of differences. Results are described as well as presented visually in graphs.

- 1. The TEC numeracy assessment raw data was available in Excel spreadsheet form and identified the cluster of questions each learner received. The clusters were separated for each learner and nine possible responses were measured:
 - i. Correct response to a non-multi choice question
 - ii. Incorrect response to a non-multi choice question
 - iii. Option 'A' for a multi choice question
 - iv. Option 'B' for a multi choice question
 - v. Option 'C' for a multi choice question
 - vi. Option 'D' for a multi choice question
 - vii. Option 'E' for a multi choice question
 - viii. Option 'F' for a multi choice question
 - ix. Submit with no response

- 2. The question bank was also sorted by Progression at each Step. Actual count totals, descriptive statistics of percentages and means were formulated from the data, and graphs and tables were then generated.
- 3. The employment data was analysed using descriptive statistics of percentages and thematic analysis of qualitative data.

Section 3: Results

Demographic Data

Numbers of Learners

The final data set comprised 176696 learner assessments with 3.635649 million responses to questions. There were 11% more learner assessments in 2016 (n=93034) than 2017 (n=83662). Figure 1 shows the total number of learner assessments.

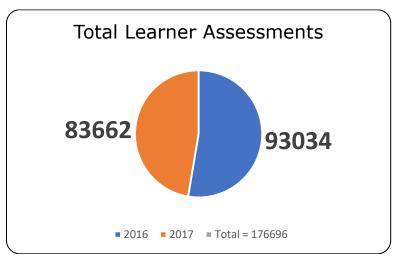


Figure 1: Total Learner Assessments

The number of learners in each of the numeracy Steps for 2016 and 2017 can be found in Figure 2. In 2016 and 2017 there were similar percentages of students in each Step. Step 4 had the highest number of students in both 2016 and 2017 with 34% (n=31770) and 33.5% (n=28124) of total learners respectively. Learners in Step 5 comprised the second largest numbers with 30.5% (n=28520) in 2016 and 31% (n=25903) in 2017. Twenty percent of the 2016 (n=18785) and 2017 (n=16943) learners were on Step 3 and 10% of learners in 2016 (n=9407) and 2017 (n=8495) were on Step 2. The lowest number of learners were on Step 1 with 5% in each of 2016 (n=4552) and 2017 (n=4197).

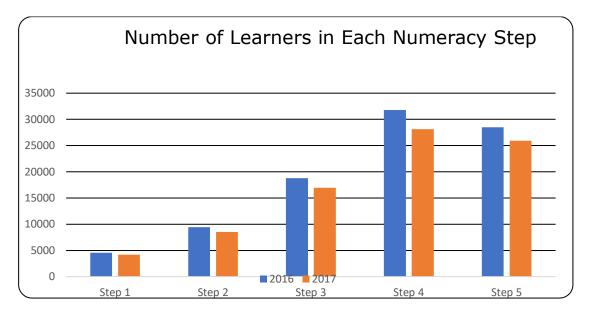


Figure 2: Number of Learners in Each Numeracy Step

Gender

Gender distribution across 2016 and 2017 can be found in Figure 3 and identifies mean gender differences. Although the data identifies a higher female percentage representation in Steps Two, Three and Four, the differences are not significant.

```
Step 1: Females 48% (2016) and 46% (2017) - mean is 47%<br/>Males 52% (2016) and 54% (2017) - mean is 53%There are 6% more males than females in Step 1 across both years (p=0.55)Step 2: Females 51.5% (2016) and 53.5% (2017) - mean is 52.5%<br/>Males 48.5 (2016) and 46.5 (2017) - mean is 47.5%There are 5% more females than males in Step 1 across both years (p=0.62)Step 3: Females 54.5% (2016) and 55% (2017) - mean is 55%<br/>Males 45.5% (2016) and 55% (2017) - mean is 45%There are 10% more females than males at Step 3 across both years (p=0.32)Step 4: Females 54% (2016) and 55% (2017) - mean is 45.5%There are 9% more females than males at Step 4 across both years (p=0.37)Step 5: Females 49% (2016) and 50% (2017) - mean is 49.5%Males 51% (2016) and 50% (2017) - mean is 50.5%
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There are 1% more males than females at Step 5 across both years (p=0.92)

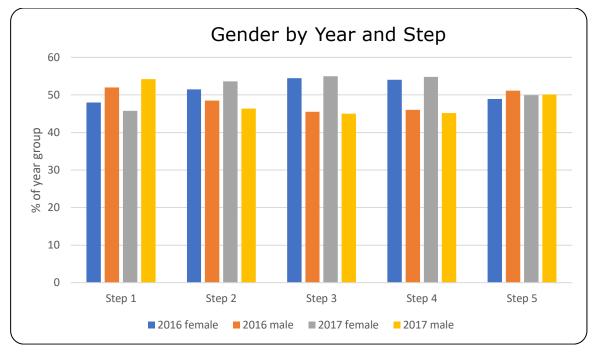


Figure 3: Gender by Year and Step

Age

All Steps identify at least a 100% increase in numbers of Under-18 year old learners from 2016 to 2017 (see Figure 4). Eight percent of 2016 learners in Step 1 were Under-18, increasing to 17% in 2017 (112.5% increase). Eleven percent of 2016 learners in Step 2 were Under-18, increasing to 23% in 2017 (109% increase). In Step 3, the Under-18 age group comprised 10% in 2016 and 20% in 2017 (100% increase). In 2016 8% of Step 4 learners were Under-18 years of age, increasing to 18% in 2017 (125% increase). Step 5 Under-18 years age group was 5% of the cohort rising to 12% in 2017 (140% increase).

The 18-19 year age group population altered slightly and insignificantly downwards in numbers between 2016 and 2017. In Step 1, there was a slight drop from 17% to 16% (decrease 1%, p=0.86); Step 2 from 23% to 19% (decrease 4%, p=0.54); Step 3 from 23% to 19% (decrease 4%, p=0.54); Step 4 from 22% to 19% (decrease 3%, p=0.64) and Step 5 stayed at 18% for both years.

The 20-24 age group numbers consistently fell between 2016 and 2017 across all the Steps, although individual changes were not statistically significant. In Step 1, the percentage dropped from 21% to 16% (decrease 5%, p=0.41). In both Step 2 and Step 3, the percentage dropped from 19% to 15% (decrease 4%, p=0.49). In Step 4 the percentage changed from 24% to 16% (decrease 8%, p=0.21) and in Step 5 from 24% to 17% (decrease 7%, p=0.27).

The Over-40 year age group comprises the largest consistent age bracket across the Steps. The Step 1 mean is 29% (30% in 2016 and 28% in 2017), 22% in Step 2 (22% in 2016 and 2017), 23.5% in Step 3 (24% in 2016 and 23% in 2017), 21.5% in Step 4 (21% in 2016 and 22% in 2017) and finally 24% in Step 5 (24% in 2106 and 24% in 2017).

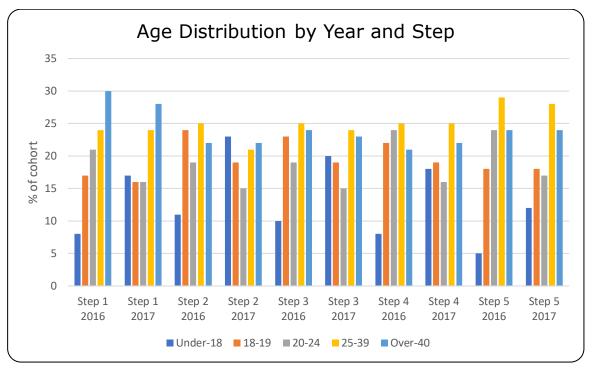


Figure 4: Age Distribution by Year and Step

Ethnicity

The 2016 and 2017 cohorts have been combined for analysis of the ethnicity data. For the purposes of this project only learners indicating Māori, New Zealand European Pākehā (EPK) or one of the ethnicity options that pre-combined to Pasifika have been analysed. See Figure 5 for the ethnicity distribution by Step.

Overall the data trend shows that the New Zealand European Pākehā representation increases significantly by 50% (p=0.05) through the Steps from 37% in Step 1 to 55.5% in Step 5. Māori trend slightly decreases by 7% (p=0.4) from 39% in Step 1 to 32% in Step 5. The Pasifika trend line decreases significantly by 45.5% (p=0.06) from 23.5% in Step 1 to 12.5% in Step 5.

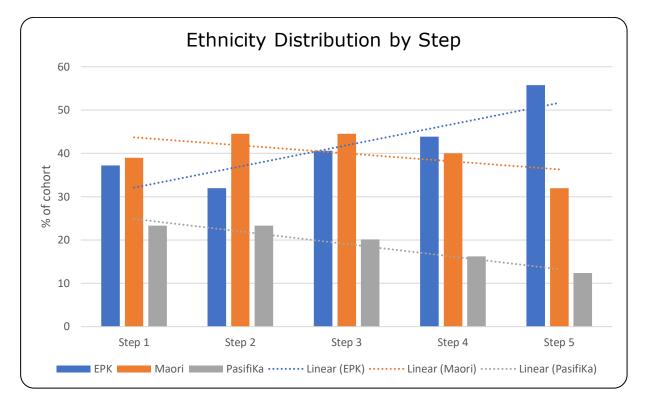


Figure 5: Ethnicity Distribution by Step

English as Second Language

The 2016 and 2017 cohorts have been combined for analysis of those students who identified having 'English for speakers of other languages' (ESOL). Figure 6 shows the number of students overall who identified as an ESOL student in each Step. There is an inverted bell-shape of the curve across the Steps with 26.5% of ESOL students in Step 1, 22% at Step 2, 20.5% in Step 3, 19.5% in Step 4 and 23% in Step 5.

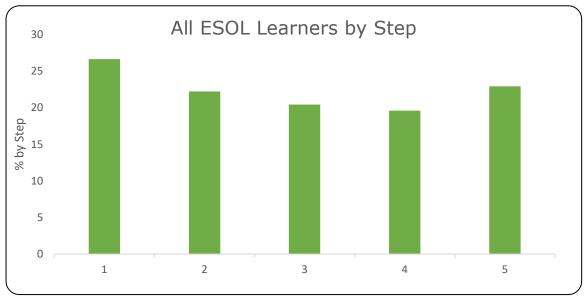


Figure 6: All ESOL Learners by Step

Figure 7 shows ESOL learners by ethnicity and Step. At each Step the number of New Zealand European Pākehā learners indicating English is not their first language increases from 6.5% in Step 1 to 13.5% in Step 5. This is slightly greater than Māori learners indicating English is not their first language where 5.5% indicated in Step 1 to 9.5% in Step 5. The percentages of Pasifika learners indicating English is not their first language is 53% at Step One, reducing to 38% at Step Four then increasing to 39.5% at Step Five.

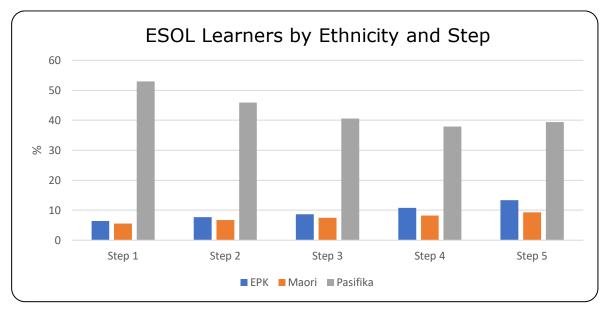


Figure 7: ESOL Learners by Ethnicity and Step

Numeracy Assessment Data

Time Taken to Complete Numeracy Tests

The mean length of time that students took to undertake their numeracy test was measured in minutes, data for 2016 and 2017 can be seen in Figure 8. Those students taking less than four minutes or over 4 hours were excluded. There were little differences found between 2016 and 2017 although learners all took slightly longer in 2017 than 2016, with the exception of Step 1 where the time was the same - 21.5 minutes. The mean times were, Step 2: 2016 - 25.5 minutes and 2017 – 27 minutes; Step 3: 2016 - 27.5 minutes and 2017 - 29.5 minutes; Step 4: 2016 - 29.5 minutes and 2017 – 31 minutes; Step 5: 2016 - 31.5 minutes and 2017 – 33 minutes.

The amount of time taken to complete the numeracy assessments increased as the Steps progressed. The mean time in Step 1 is 21.5 minutes, Step 2 is 26.25 minutes, Step 3 is 28.5 minutes, Step 4 is 30.25 minutes and Step 5 is 32.5 minutes. Although there is a 51% increase in time taken from Step 1 to Step 5, the difference is not significant (p=0.15).

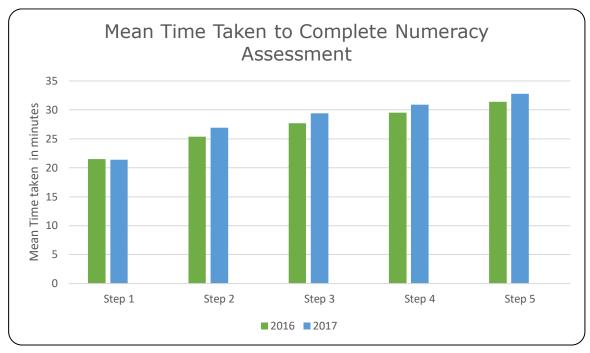
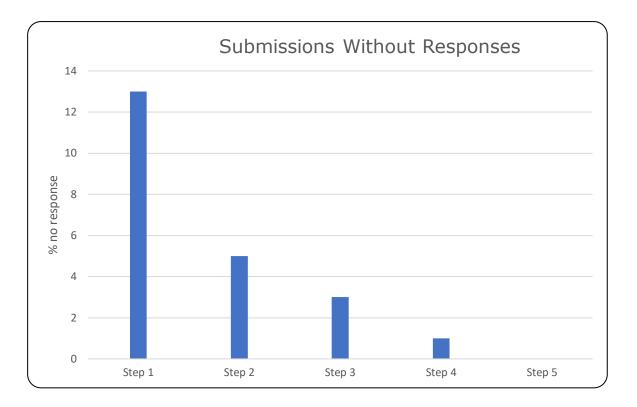
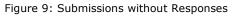


Figure 8: Mean Time Taken to Complete Numeracy Assessment

Test 'No Responses' and Errors

The number of questions across the Steps, submitted without offering a response can be found in Figure 9. The highest number was in Step 1 where 13% (n=23553) did not offer a response. The percentage drops away considerably over the next Steps with 5% (n=17198) in Step 2, 3% (n=18701) in Step 3, 1% (n=12687) in Step 4 and 0.5 % (n=4302) in Step 5.





Errors in the 3.6 million learner responses to questions in the LNAAT numeracy assessments were grouped into progressions for each Step. The seven numeracy progressions were filtered, and counts have been made on:

- Correct and incorrect responses for non-multi-choice questions.
- Correct and incorrect responses for multi choice questions and the incidence of each incorrect multi-choice option.
- Questions learners did not offer a response to but continued with the assessment were deemed neither correct nor incorrect

Results can be found in Figure 10.

In Step 1 (n=176301), error rates were 68.5% in Proportional Reasoning, 69% in Multiplicative Strategies, 60.5% in Measurement, 64.5% in Place Value, 63.5% in Additive Strategies, 62.5% in Number Facts, and 56% in Number Sequence. The mean error rate in Step 1 is 63.5%.

In Step 2 (n=378047), error rates were 61% in Proportional Reasoning, 59% in Multiplicative Strategies, 54% in Measurement, 51.5% in Place Value, 53.5% in Additive Strategies, 52% in Number Facts, and 47.5% in Number Sequence. The mean error rate in Step 2 is 54%.

In Step 3 (n=731790), error rates were 56% in Proportional Reasoning, 50% in Multiplicative Strategies, 50% in Measurement, 48% in Place Value, 46% in Additive Strategies, 44.5% in Number Facts, and 43.5% in Number Sequence. The mean error rate in Step 3 is 48%.

In Step 4 (n=1200833), error rates were 47% in Proportional Reasoning, 43.5% in Multiplicative Strategies, 46% in Measurement, 42% in Place Value, 36% in Additive Strategies, 38% in Number Facts, and 39% in Number Sequence. The mean error rate in Step 4 is 41.5%.

In Step 5 (n=1048678), error rates were 38.5% in Proportional Reasoning, 32.5% in Multiplicative Strategies, 38% in Measurement, 33.5% in Place Value, 34% in Additive Strategies, 24.5% in Number Facts, and 33.5% in Number Sequence. The mean error rate in Step 5 is 33.5%.

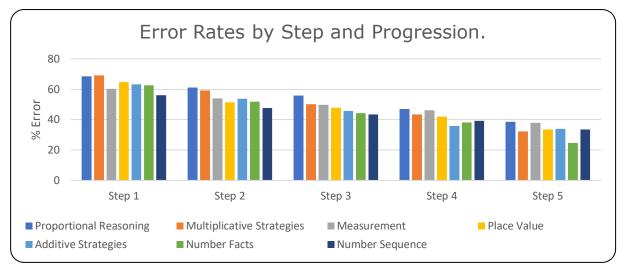


Figure 10: Error Rates by Step and Progression

The all-progression error rate means fell steadily from Step 1 at 63.5% to Step 5 at 33.5%. The Proportional Reasoning progression has the highest rate of learner error from Step Two to Step Five. At Step One, Proportional Reasoning is second to Multiplicative Strategies by just 0.5%. The lowest error rate was demonstrated in Number Sequence in Steps One to Three. Table 2 shows the rankings of progression error rates across the Steps.

	Step 1	Step 2	Step 3	Step 4	Step 5
	Multiplicative Strategies 69%	Proportional Reasoning 61%	Proportional Reasoning 56%	Proportional Reasoning 47%	Proportional Reasoning 38.5%
	Proportional Reasoning 68.5%	Multiplicative Strategies 59%	Multiplicative Strategies 50%	Measurement 46%	Measurement 38%
_	Place Value 64.5%	Measurement 54%	Measurement 50%	Multiplicative Strategies 43.5%	Additive Strategies 34%
	Additive Strategies 63.5%	Additive Strategies 53.5%	Place Value 48%	Place Value 42%	Place Value 33.5%
$\mathbf{\mathbf{V}}$	Number Facts 62.5%	Number Facts 52%	Additive Strategies 46%	Number Sequence 39%	Number Sequence 33.5%
	Measurement 60.5%	Place Value 51.5%	Number Facts 44.5%	Number Facts 38%	Multiplicative Strategies 32.5%
	Number Sequence 56%	Number Sequence 47.5%	Number Sequence 43.5%	Additive Strategies 36%	Number Facts 24.5%

Table 2: The Rankings of Progression Error Rates across the Steps

The mean percentages error scores across Steps 1-5 was 54% for Proportional Reasoning, 51% for Multiplicative Strategies, 49.5% for Measurement, 48% for Place Value, 46.5% for Additive Strategies, 44.5% for Number Facts and 44% for Number Sequence as shown in Figure 11.

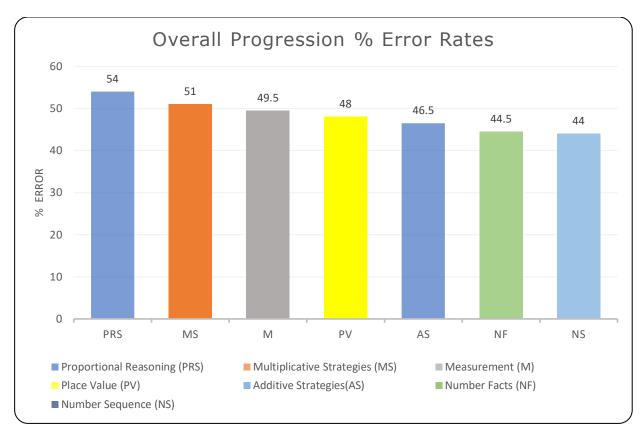


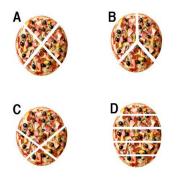
Figure 11: Overall Progression Percentage Error Rates

Proportional Reasoning

The highest error rate is Proportional Reasoning in each Step except Step 1 where it is lower than Multiplicative Strategies by only 0.5%. It also has the highest mean score across all the Steps at 54%. The LNAAT question database contains 209 questions directly related to Proportional Reasoning. Three examples of LNAAT Proportional Reasoning Strategy questions are further examined. In all three example questions, learners were able to use a calculator to assist with answering.

Images of Pizza

This question has the least difficulty for learners. Figure 12 shows the question. It was proposed 1085 times and has an overall error rate of 17.5%. Step One learners had an error rate of 30% and this declined to 0% for Step 5 learners. This was not a multi-choice question.



Pizza Cut Huhanna shares a pizza equally between 4 people. Click on the picture that shows how Huhanna cuts her pizza.

Figure 12: Images of Pizza Question

Membership Fee

The following question has a medium difficulty level and comprises a multi-choice answer.

B	B
	mal
Member	ship Fee
\$3	20
Specia	Offer
20%	OFF
	Aoney

Membership fee – Special offer	
How much money will you save?	

...

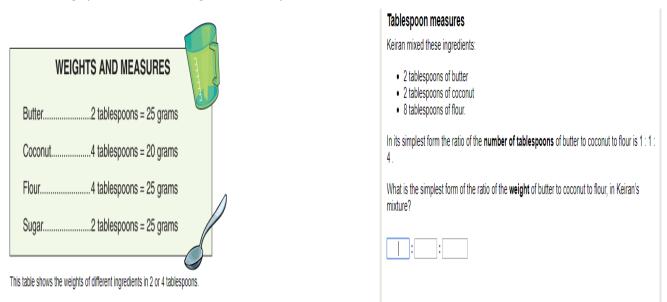
◯ \$20	
\$32	
○ \$60	
\$64	

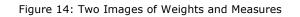
Figure 13: Membership Question

The correct answer was the fourth option (\$64) attained by 6%. The first option of \$20.00 was the most common response. The second response (\$32) was chosen by 21% of the learners, and the third response (\$60) was chosen by 23% of learners. Twenty percent of learners opted for no response.

Two Images of Weights and Measures

The following question has the highest difficulty level.





This guestion was proposed 12,449 times and has an incorrect response of 93% and a no response of 3.9%.

Skills Highway Employment Data

Skills Highway provided data related to employment numeracy requirements. Question 1 related to the LNAAT numeracy progressions. The stem of the seven questions started with 'Do you need entry-level workers who can...'? Using a 4-point Likert scale, employers were asked to rate the level of skill required for each progression question from not important to crucial. Figure 15 shows the percentage of employers who rated each question at each level of the scale.

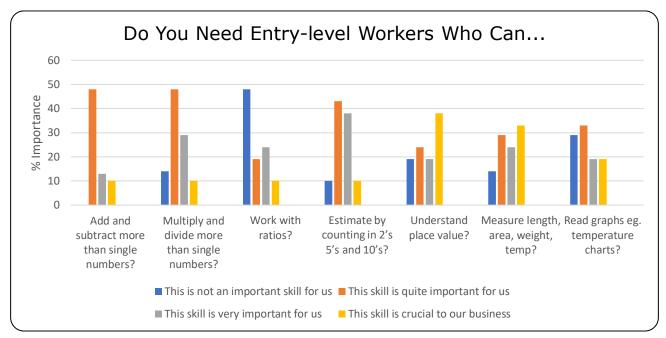


Figure 15: Employers Rating of Numeracy Skills Required

Question 3 asked employers to identify entry-level workers literacy skills gaps under the following headings.

- 1. Add and subtract more than single numbers (Additive Strategy Step 3): 48% of employers rated this as a quite important skill (13% very important, 10% crucial and 0% not important). Nineteen percent thought most entry-level workers struggle but 71% perceived most entry-level workers have acceptable skills and 10% good skills.
- 2. Multiply and divide more than single numbers (Multiplicative Strategy Step 4): 48% of employers rated this as a quite important skill (29% very important, 10% crucial and 14% not important). Forty-five percent of employers thought most entry-level workers struggle, but equally, 45% perceived most entry-level workers have acceptable skills. Five percent do not need these skills and 5% perceived entry-level workers had good skills.
- 3. Work with ratios (Proportional Reasoning Step 6): 24% of employers rated this as a very important skill (19% quite important, 10% crucial and 48% not important). Thirty-three percent of employers do not require this skill. Fifty-eight percent thought that entry-level workers struggle and 10% have acceptable skills.
- 4. Estimate by counting in 2's, 5's and 10's (Number Facts Steps 1 and 2): 43% of employers rated this as a quite important skill (38% very important, 10% crucial and 10% not important). Sixty-two percent of employers perceived acceptable entry-level worker skills with 24% perceiving entry-level workers struggle. Ten percent saw good skills in this area whilst 5% did not require these skills.
- 5. Understand place value (Place Value Steps 2-5): 38% of employers rated this as a crucial skill (24% quite important, 19% very important and 19% not important). Fifty-five percent of employers thought entry-level workers struggled and 30% thought workers had acceptable skills. Fifteen percent of employers did not need this skill and no employer perceived a good level of skills.

- 6. Measure length, area, weight, temperature (Measurement Steps 1-6): 33% of employers rated this as a crucial skill (29% quite important, 24% very important and 14% not important). Forty-eight percent of employers saw entry-level workers struggle with this skill with 33% perceiving acceptable level of skill. No employer thought workers had a good level of skill and 19% did not need the skill.
- 7. Read graphs e.g.: temperature charts (Analysing Data for Interpretation Steps 1-6): 33% of employers rated this as a quite important skill (19% very important, 19% crucial and 29% not important). Forty-eight percent of employers thought entry-level workers struggled, with 33% perceiving an adequate skill level. There were no employers perceiving a good skill level and 19% did not need this skill.

Figure 16 shows that overall, entry-level workers are not perceived to have a good level of numeracy skills. In five of the seven skills, the highest percentages among the 4 options was workers struggled. The other two skills were seen as demonstrated at an acceptable level only.

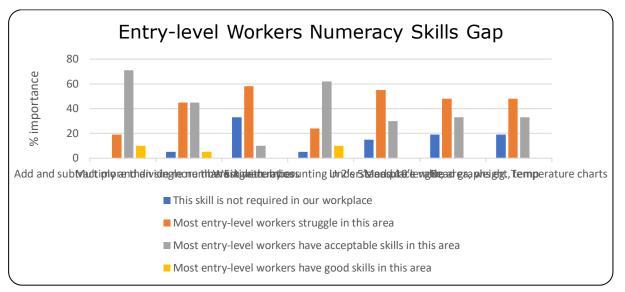


Figure 16: Entry-level Workers Numeracy Skills Gaps

Question 2 asked if there were any other numeracy skills considered important in the workplace. Table 3 shows the results along with where these skills sit in the numeracy progressions.

Important Workplace Numeracy Skills	Associated Numeracy Progression
Understanding the difference made to a number with the placement of a decimal point	Number Sequence Step Four
Time sheeting	Measurement Steps Two - Three
Working out areas	Measurement Steps One - Six
Transposing figures causes issues	No numeracy progression aligns with this issue
Keeping a balance sheet for petty cash	Additive and Multiplicative Steps One Six
Interpreting tables, measuring volumes	Measurement Step Six
Angles	Measurement Steps Two - Six
Geometry and angles.	Measurement Steps Two - Six

Table 3: Other important numeracy skills in the workplace

Finally, employers were asked if they had any other comments about entry-level workers numeracy skills. The comments can be found in Table 4 and the key themes that emerged are that entry-level workers are younger, have learned to hide their numeracy deficits, and employers expressing some frustration that these deficits are not addressed prior to employment.

- Entry-level workers are often (although not exclusively) younger workers. They
 are usually very familiar with using digital technology and gamification of
 numeracy learning would be a natural and comfortable environment for them.
 These kinds of methodologies, by their nature, provide individualised feedback and
 opportunities for repetition and (psychological) reward.
- The majority of people we employ have usually struggled at school and are relatively adept at hiding numeracy and literacy issues. This is why the program is important for us because we usually involve new staff members in the program to assist ourselves and them to understand the actual level of understanding in a non-judgemental way. This assists both parties.
- Most of our workers were assessed at L1/2 even those straight from school. Why isn't this being addressed before it becomes a workplace problem?
- Different roles have different requirements e.g. ratios and fractions are crucial for Registered Nurses
- It would be great if there was an assessment available to employers that linked directly the TEC to highlight funding needs upfront rather than having to wait until we have engaged with a provider.

Table 4: Employer comments regarding entry-level workers literacy skills

The responses to working with ratios, percentages and fractions in questions one to three are Proportional Reasoning abilities and align with the tertiary level deficit identified by this research. Question one rates these skills as 'being needed' in new employees, and is the area identified as having the most workers struggling in the area in question three. Measurement of length, area, weight, and temperature, reading and interpreting graphs and statistics, multi-digit multiplication and place value, are identified as being a struggle for entry-level employees. Proportional Reasoning, although not indicated as the most important skill needed by entry-level workers, is listed as the highest observed area of skill gap once employed. Place Value and Measurement also feature in both sets of results. Table 5 compares the LNAAT Numeracy Progression error rates from highest to lowest with the employers' perception of numeracy skill gaps from highest to lowest.

LNAAT literacy progressions greatest error rates	Employers view of skills gaps
Proportional Reasoning	Proportional Reasoning
Multiplicative Strategies	Place Value
Measurement	Measurement
Place Value	Graphs and Statistics
Additive Strategies	Multiplicative Strategies
Number Facts	Additive Strategies
Number Sequence	

Table 5: Rating comparisons between numeracy progression errors and workplace numeracy skills gaps

Section 4: Discussion

Demographics

There were 176696 learner assessments analysed in this research project with a 10% increase from 2016 to 2017. The biggest changes were the increase in the numbers of Under-18 year old learners. In each Step, there was at least 100% increase in numbers. The Tertiary Education Commission development of programmes such as the Secondary Tertiary Pathways, 3 + 2 and Dual Pathways programmes engaged secondary learners with tertiary providers. The combination of these may explain the increase in numbers as the Ministry of Education has encouraged schools to use the LNAAT to support or inform particular accelerated literacy and numeracy interventions offered within secondary schools.

Although there were slight decreases in numbers of learners from 2016 to 2017 in the 18-19 year age group, the Under-18 and 18-19 year old cohorts are the groups most recently exposed to secondary school education and therefore would most likely have had recent numeracy education. This combined age group comprised 25% of Under-18 and 33% of 18-19 year old learners in Step 1. The numeracy deficits may indicate a shortfall in numeracy content from the secondary sector resulting in a lack of understanding of these learners.

The low educational achievement rates of Māori and Pasifika learners is widely discussed in education and society. Although there is only a slight decrease in Māori representation from Step 1 to Step 5, there is a significant decrease for Pasifika and significant increase for New Zealand European Pākehā. There is no evidence in this research that identifies a bias to any numeracy achievement Step for Māori learners. That implies that there is no evidence of skill deficit that would prevent a Māori learning and achieving in numeracy. In spite of a dominant discourse that underlines Māori learner underachievement, this research highlights that Māori learner numeracy step achievement in LNAAT is evenly distributed across the learning Steps. Pasifika learners are depicted as an inverse bell curve across the numeracy achievement Steps.

A quarter of learners at Step 1 and Step 2 do not have English as their first language giving rise to the question of the impact of literacy on numeracy skills. However, just less than a quarter of learners at Step 5 and Step 6 also do not have English as their first language and yet achieve good numeracy outcomes. The significance is apparent when considering ethnicity. Over half of the learners at Step 1 are Pasifika and are also four times more represented at Steps 2 to Step 5 than Māori and New Zealand European Pākehā.

Numeracy Assessment Data

The length of time taken to achieve correct answers dispels the myth that 'to be good at maths, you must be fast at maths'. Those learners who achieved more highly took longer to calculate the answers. Learners therefore require time to successfully undertake numeracy exercises. The pace at which maths classes are conducted has the potential to significantly influence the engagement of learners with the topic. Messages from learners of being 'left behind' and 'switching off' are common. 'The same people answer all the questions, so I just sit there' is a common response in the classroom. The research data identifies the time taken with a numeracy assessment is directly related to the Step achieved. This idea of quality verses quantity is not new, but basic good teaching practice. The ability to slow numeracy classes to allow learners to calculate solutions rather than nodding in agreeance with a minority of learners who have the ability to rapidly offer a solution and move the lesson forward is essential to keep the majority engaged and achieving.

Learners have the option of not responding to questions. This was greatest in Step 1 at 13% and 5% in Step 2. The numbers sharply fell away in Steps 3 to Step 5. The 'no responses' at Step 1 and Step 2 raise questions on whether this is due to a lack of numeracy skills or low literacy skills reducing the ability of the learner in understanding the text of the question.

The Proportional Reasoning progression is identified as having the highest rate of learner error from Step Two to Step Five except Step 1 where it is second to Multiplicative Strategies by only 0.5%. It also has the highest mean score across all the Steps at 54%. The LNAAT numeracy question bank has fewer Proportional Reasoning questions for the lower Steps as the skills of Proportional Reasoning require base numeracy knowledge that is covered by other progressions, and this may explain the close second of Proportional Reasoning at Step 1.

Proportional Reasoning has been called the backbone, the cornerstone, the gateway to higher levels of mathematics success, and is considered as a "capstone" of primary school mathematics (Kilpatrick, Swafford & Findell, 2001; Lamon, 1999; Lesh, Post & Behr, 1988; as cited in Parish, 2015). Proportional Reasoning involves "making multiplicative comparisons between quantities" (Wright, 2005; as cited in Parish, 2015), together with "the ability to mentally store and process several pieces of information" (Lesh, et al, 1988, p. 469; as cited in Parish, 2015). According to Lamon (1999), "Proportional Reasoning is one of the best indicators that a student has attained understanding of rational numbers" (as cited in Parish, 2015).

Proportional Reasoning is a lynchpin for future success in mathematics and science (Lesh, Post, & Behr, 1988; as cited in Carney, Smith, Hughes, Brendefur, & Crawford, 2015). Based on a substantial body of Proportional Reasoning research (Lamon, 2005; Lobato, Ellis, & Charles, 2010,; as cited in Carney, Smith, Hughes, Brendefur, & Crawford, 2015), there have been several calls for shifting instruction from the typical focus on the cross-multiplication algorithm to students' meaningful understanding and application of ratio related concepts (National Governors Association & Chief Council of State School Officers, 2011). However, implementing this shift in instruction is difficult. Schools and teachers need resources to support this change and more information is needed on how students' Proportional Reasoning develops from less efficient to more efficient strategies. There appears to be a sense of general lack of knowledge of the skills of Proportional Reasoning. The talk about addition, subtraction, multiplication, division, percentages, and ratios finds a base understanding of these concepts. However, mention of Proportional Reasoning brought comments like 'what's that'? The term and its meanings appear less commonly understood as a concept.

Significant error rates were also found in other progressions. Approximately half of learners had errors in Multiplicative Strategies, Measurement and Place Value. Slightly less than half had errors in Additive Strategies, Number Facts and Number Sequence. If a conservative 40% general numeracy error rate is suggested, this must be a concern for employment and everyday life.

Skills Highway Employment Data

Proportional reasoning skills have been identified as the greatest number of learner errors as well as the skill that employers have observed having the biggest entry-worker gap. Traditional tertiary tradebased training, (e.g.: Automotive, Engineering, Building, Hairdressing and Hospitality) have aspects of Proportional Reasoning, from hair colouring to two stroke petrol mix to concrete mixing. The ability of learners to have a strong understanding of this numeracy concept is fundamental to their vocations, yet the data identifies Proportional Reasoning as most lacking. The issue of numeracy deficit in Proportional Reasoning must be addressed in the fundamental learnings of numeracy. Therefore, overt inclusion of Proportional Reasoning in primary and secondary curricula requires a longer and stronger focus. Two New Zealand based resources along with additional Ministry support, could boost the development of suitable interactive material for learners. These are:

- Teaching adults to make sense of number to solve problems. (Pages 58 to 65) (Tertiary Education Commission, 2008b)
- Pathways Awarua Pathways offers a teacher/ tutor/ learner resource and is the ideal platform to further develop step-by-step interactive Proportional Reasoning resources for learners.

Educational Approaches

The research identified a number of short exercises and single lessons focusing on Proportional Reasoning however, there is a lack of a systematic resource that introduces and builds on Proportional Reasoning skills. Pathways Awarua is the ideal platform for such a resource.

The National Governors Association & Chief Council of State School Offices (2011) state that "there have been several calls for shifting instruction from the typical focus on the cross-multiplication algorithm to students' meaningful understanding and application of ratio related concepts". However, implementing this shift in instruction is difficult. Schools and teachers need resources to support this change and more information is needed on how students' Proportional Reasoning develops from less efficient to more efficient strategies.(As cited in Carney, Smith, Hughes, Brendefur, and Crawford, 2015). For the shift from a focus on the cross-multiplication algorithm to significant perception and use of ratio related concepts to occur, "we must modify or adapt our numeracy curriculum to ensure learners have the proportional reasoning numeracy skills necessary for their professions." (Carney, Smith Hughes, Brendefur, & Crawford, 2015, p. 141).

Section 5: Recommendations

From the analysis of results and discussion of the findings, the following recommendations are made regarding literacy.

- The development of a specific staircase Proportional Reasoning module in Pathways Awarua.
- Professional development for tutors in presentation of Proportional Reasoning delivery material.
- The availability of a 'read to me' option for numeracy assessments in the LNAAT

The introduction of a 'read to me' capability in LNAAT numeracy would give the opportunity to negate or lessen the influence of low reading skills in assessed learners. A pilot using a 'read to me' function as part of the numeracy assessment with a random selection of ' low numeracy low reading skill' learners would result in meaningful data regarding the influence of reading on numeracy ability.

Further research is recommended in the following areas.

- 1. Literacy/reading: a similar analysis of literacy achievement among learners and from employers
- 2. A comparative study between numeracy and literacy Step achievements for a selection of learners to give a greater knowledge base on:
- 'low numeracy high reading skills' learners
- `high numeracy low reading skills' learners
- `low numeracy low reading skills' learners
- high numeracy high reading skills' learners

The influence of Literacy Skills in Numeracy and vice versa is key in any further analysis of numeracy skills, the ability of being able to analyse how reading skills have influenced a response to a numeracy question will allow a distinction between low numeracy skills and low numeracy skills as a result of low reading skills. The ability to analyse the LNAAT reading assessment data for the same sample years then link the numeracy and reading results on a per Step or per learner basis is a crucial next step in this research. A common example is learner 'K', a Step Five numeracy learner who placed a dollar symbol in her answer where the input screen had already contained the dollar symbol, this means 'K's' answer was deemed incorrect whereas the actual numeracy content was correct.

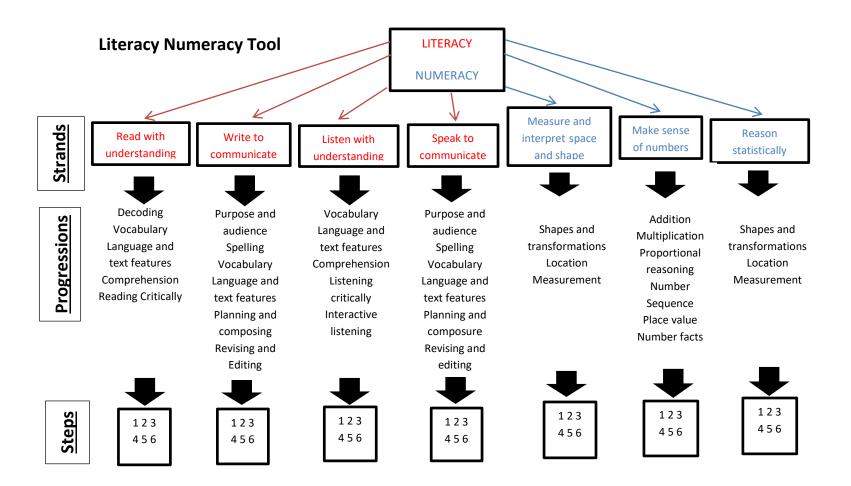
- 3. An exploratory study conducted within the primary and secondary sector, focusing on Proportional Reasoning, asking:
- how it is introduced to learners,
- effective styles of teaching and presentation of proportional reasoning
- and the use of contextual Proportional Reasoning problems in the classroom

Identifying best practice teaching methods to inform process and curriculum modifications would positively advance engage students with proportional reasoning.

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Appendix One The LNAAT Assessment Tool (TEC, 2008a & 2008b)



Appendix Two The Learning Progressions

Progressions	Steps
Additive	Solve addition and subtraction problems by counting all objects.
Strategies	Solve addition and subtraction problems by counting on or counting back using ones and tens.
et ategiet	Solve two-digit addition and subtraction problems mentally, using partitioning strategies.
	Solve multi digit addition and subtraction problems using partitioning strategies.
	Or alternatively
	Justify the reasonableness of answers to problems solved, using a calculator or algorithm.
	Solve addition and subtraction problems involving decimals and integers, using partitioning strategies.
	Or alternatively
	Justify the reasonableness of answers to problems solved, using a calculator or algorithm.
Measurement	Compare and order objects directly, using attributes of length, area, volume, and capacity, weight, angle,
	temperature, and time intervals in order to understand the attributes.
	Use repetition of a single unit to measure length, area, volume, and capacity, weight, angle, temperature, and time.
	Use repetition of a single unit to measure length, area, volume and capacity, weight, angle, temperature and time.
	Select and use sensible units (both informal and standard or formal units) to measure length, area, volume and
	capacity, weight, angle, temperature, power and time.
	Use common benchmarks to select appropriate methods for estimating measurements carry out simple unit
	conversions within a measurement system.
	Select use sensible units and tools and / or formulas to measure the side lengths, perimeters and areas of rectangles,
	circles and triangles to appropriate levels of precision.
	Carry out conversions within a measurement system.
Multiplicative	Solve multiplication problems by counting the objects.
Strategies	Solve multiplication problems by skip - counting, often in conjunction with one - to - one counting and often keeping
	track of the repeated counts by using materials (for example, fingers) or mental images.
	Solve single-digit multiplication and division. Problems mentally, using known multiplication facts and repeated
	addition.
	Solve multiplication and division problems with single-digit multipliers or divisors mentally, using partitioning strategie
	and deriving from known multiplication facts.
	Solve multiplication or division problems with multi-digit whole numbers, using partitioning strategies.
	Or alternatively
N / 5 /	Justify the reasonableness of answers to problems solved, using a calculator or algorithm.
Number Facts	Addition facts that sums of 5 or 10 and the decade facts.
	Basic addition and subtraction facts up to $10 + 10$.
	Basic multiplication and division facts up to 10×10 .
	Basic multiplication facts with tens, hundreds and thousands.
	Fraction and decimal groupings that make 1. Common factors of numbers up to 100.
	Fraction, decimal and percentage conversions for $1/2$, $1/3$, $1/4$, $1/5$, $1/10$
N/	The convention for exponents.
Number	The sequence of numbers forwards and backwards to at least 20.
Sequence	The sequence of numbers forwards and backwards, to at least 100.
	The sequence of numbers, forwards and backwards, to at least 1,000.
	The number that is 1, 10 and 100 before or after a given number in the range 0 – 1,000.
	How to skip - count in twos, threes, fives and tens to 1,000.
	How to order fractions with like denominators.
	The sequence of numbers, forwards and backwards, by ones, tens, hundreds and thousands, to a million. How to give the number 1, 10, 100 or 1,000 before or after a given number in the range 0 – 1,000.000.
	The sequence of decimal numbers in tenths and hundredths.
	How to order unit fractions.
	The sequence of integers, fractions, decimals and percentages, forwards and backwards, from any given number.
Proportional	There is a gap at the first Step of this progression because learners need to be able to count all objects before they
Reasoning	can use the strategy of equal sharing.
ncasoning	Find a fraction of a set by using equal sharing.
	There is a gap at the third Step of this progression because learners need to know single - digit multiplication and
	division facts before they can use them to find fractions of whole numbers.
	Use known multiplication and division facts to find fractions of a whole number.
	Use multiplication and division strategies to solve problems that involve simple equivalent fractions and simple
	Use multiplication and division strategies to solve problems that involve simple equivalent fractions and simple conversions between fractions, decimals and percentages.
Place Value	conversions between fractions, decimals and percentages.
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Place Value	 conversions between fractions, decimals and percentages. There is a gap at the first Step of this progression because learners need to be able to count all objects before they can learn to count in tens. 10 as a counting unit, the tens in numbers to 10 and the place values of digits in whole numbers up to 100. The tens and hundreds in numbers to 1,000 and the place values of digits in whole numbers up to 1,000. How many tens, hundreds and thousands there are in any whole number.